Playing Checkers in Chinatown*

José-Antonio Espín-Sánchez and Santiago Truffa
Yale University — ESE, Universidad de los Andes

[VERY PRELIMINARY, PLEASE DO NOT CITE]

Abstract

In 1905-1935, the city of Los Angeles bought the water and land rights of the Owens Valley farmers and built an aqueduct to transfer the water to the city. The dark story is that Los Angeles bullied and isolated reluctant farmers to get cheap water. A map of the farmers´ plots sold in any given point in time, however, would look like a checkerboard either because the city is intentionally targeting specific farmers, whose sale would create negative externalities in the remaining farmers, or because the farmers were heterogeneous. We analyze the bargaining between the city and the farmers and evaluate the effects that farmers' actions had on one another, to assess the checkerboarding claim. We estimate a dynamic structural model of the farmers' decision on selling to the city. We found that there are large externalities when farmers sold. The externalities were larger for neighboring farmers, and when the selling farmer was closer to the river.

JEL Codes: N52, Q25, C73, L1

Keywords: Water Rights, War of Attrition

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"The only reason they were 'checkerboarding' was because this fellow wanted to sell out and the next one didn't."

A. A. Brierly (Owens Valley farmer), cited in Delameter (1977)

"Their efforts were focused on the key properties which controlled the points of access to the river, so that the less favorably situated ranchers inland could be cut off from their water supply if they refused to negotiate."

William Kahrl, *Water and Power* (1982)

1 Introduction

While urbanization is viewed as a critical engine of development, urban population growth can give rise to opportunities and challenges. As urbanization processes unfold, developing countries will have to deal with an unprecedented increase in the number of people moving into their cities. The United Nations predicts that the number of city dwellers will rise by 3.5 billion over the next 40 years (WHO, 2015). The movement of urban population from rural to urban environments will also entail a reallocation of natural resources, like water, to redeploy towards urban use. How can we ensure that the benefits these transfers generate are distributed evenly among urban and rural dwellers is a first-order concern for economists and policymakers.

To study this question, we revisit probably the most famous episode of water transfer in the history of the US. In 1905-1935 the city of Los Angeles (LA) purchased the water and land rights of the Owens Valley farmers, built an aqueduct to transfer the water, and changed the history of the Valley and that of water transfers forever. The city grew from 100,000 people in 1900 to 1.2 million in 1930, becoming the largest city in California and the second largest in the U.S., which would have been impossible without the water from the Owens River. Despite this achievement, the transfer has been immersed in controversy, exaggerated in the movie Chinatown, since its inception. The most serious accusations made against the city was that they were checker-boarding their land purchases, *i.e.*, that they were intentionally buying land surrounding reluctant sellers, to drive down their demanded price. In this paper, we first address the historical question of whether the city was checker-boarding and explore to what extent it might have behaved strategically in its purchases of land and water rights. We then study the bargaining that took place between the city and the farmers to assess how externalities might have affected the distributions of rents among the city and farmers.

This is a challenging question. First of all, there are data limitations as the current historical literature did not have geo-coded data of each transaction nor specific dates of when the plots were sold. Secondly, water is a very peculiar good. When analyzing water transfers, we need to account for the presence of externalities among farmers. Therefore, we cannot study transactions between farmers and the city in isolation but look at these from a collective decision perspective, where the city of LA could strategically buy out different farmers and thus affect the trade-off of the remaining farmers. Ultimately, when looking at the decision to sell and prices, our reduced form analysis will not be able to capture the rich endogenous interactions taking place between farmers, which would require a model that can flesh out the dynamic incentives that farmers faced in a bargaining game.

We build a novel and very detailed dataset, containing the exact date of each sale, the precise geo-location of each plot, as well as other characteristics: acreage, water rights, sale price, crops under cultivation. We use this new dataset first to assess whether the city did indeed checkerboard their purchases. To do so, we first show that there is a spatial correlation on the date of sale within the whole valley, which highlights the importance of having geo-coded data. We then show reduced form regressions to explain whether a farmer sold or not and at what price, in a dynamic setting. Our results suggest that there is a significant interaction taking place at the ditch level, between farmers that have sold their land and farmers that have not and that externalities imposed by selling neighbors drive these patterns.

We model the purchases made by the city as a War of Attrition (WoA) with externalities, which in practice resembles a Monopsonist strategy in the Coase conjecture (Coase, 1972). If the city could commit not to offer a larger price in the future, the city could extract all the surplus from the farmers. However, if the farmers could bargain as one, they might be able to obtain most of the surplus from the transfer. The situation is complicated by the heterogeneity of the farmers and the negative externalities they exert on others, *e.g.*, farmers whose plots are closer to the river where the ditch meets the river would produce a larger negative effect on the other farmers on the same ditch, than those farmers down the ditch. We exploit the detailed selling times to show that the shape of the hazard rate is not constant, which requires a model with valuations that change over time. Our focus is not on the interaction of the city vis-à-vis the farmers as a whole, but rather, the interaction among farmers, given the behavior of the city. We estimate the model on two steps. In the first step, we get a vector of the pseudo parameters that we recover using only exit times. In the second step, we use the estimated pseudo parameters and the probabilities they imply to estimate hedonic regressions to get a set of parameters for each fundamental farmer characteristic.

Unlike the previous literature on the topic, we have the location of each plot sold to the city. Therefore, there are questions we can answer that could not be explained without the location data. First, we can assess the externalities created by each farmer on the remaining farmers, *i.e.*, on their probability of selling and the price they would get. We find evidence that externalities depend on fundamental farmers' characteristics; for example, farmers with more water rights created substantial externalities. The same is true for farmers located closer to the river. Second, given the externalities, we could test the checker-boarding behavior of the city, *i.e.*, whether the city would offer more money to farmers that would create higher externalities, to drive down the prices for the remaining farmers. We do not find conclusive evidence that the city was checker-boarding. This is not surprising given that the offers were made by a committee intended to give each farmer a fair price for their land, not to minimize the time spent on negotiations.

Estimating a WoA with externalities and time changing valuations is challenging. In the presence of externalities, the payoff of each player is affected not only by their decisions but also by the decisions of other players. If a farmer sold at a given point, it could be because his value of waiting was low, given the "high" price offered by the city, or because the farmer expects one of his neighbors to sell soon, which would lower his continuation value. If we also consider that continuation values and the probability of selling are changing over time, separately identifying the parameters of such a model can be very demanding on the available data. We adapt the WoA game in Catepillan and Espín-Sánchez (2019) to our empirical setting and show that under general assumptions the parameters can be identified. Moreover, in equilibrium, the WoA with externalities and continuation values changing over time resemble a Proportional Hazard Rate Model, where the "shape" of the continuation value over time is a function of the selling probability. Moreover, the externalities and the direct effects of each variable can be estimated using simple linear regressions in a second stage. Finally, we use the estimated model to compute counterfactuals on what the prices paid would have been if the farmers had been able to bargain as one, or bargain as one in each ditch.

This paper relates to a rich literature in political economy studying the coordination problems associated with the overuse and depletion of natural resources, like water (Ostrom, 1962). The fact that water is a good that is both subtractable and challenging to exclude motivated a long tradition analyzing common-pool resources (Ostrom, 2010), and how to institutionally deal with the coordination problems that externalities entail. In our setting, we study how the city of Los Angeles strategically exploited this feature of waters markets, to maximize the amount of rents it could extract from the farmers.

In the presence of externalities, private decisions by farmers can be seen as a form of collective decision. Literature in vote buying (Dal Bo, 2007) has shown that a principal can influence collective decisions of agents to induce inefficient outcomes at almost no cost, as long as the principal can reward decisive players differently, and agents face high coordination costs. We show evidence consistent with the prediction that the city should target key players strategically, and that by doing so many farmers were bought at a price very close to their marginal costs.

There is extensive academic work on the Owens Valley controversy. The historical literature focuses on the characters of the story, and how their personal beliefs and personality traits affected the outcome (Hoffman, 1981; Kahrl, 1982; Davis, 1990). However, they differ on whether they portray LA as a villain, or just as a rational business-minded agent. Whereas Kahrl (1982) and Reisner (1987) portray the citizens of the valley as innocent victims, Hoffman (1981) takes a more neutral view of the situation as inevitable given the population growth of LA in the early 20^{th} century. Moreover, the few accounts that we have from Owens Valley farmers, such as Delameter (1977) and Pearce (2013), tell a different story. Their story is one of farmers willing to sell their ruinous farms, while the townspeople, with the help of the Watterson brothers and the local newspaper, bullied both the farmers and the city agents until they got compensation for their urban properties, which eventually happened in 1925. There has been some recent work in economics, most prominently by Libecap (2005, 2007, 2009). He focuses on the prices that farmers received for their lands. He showed that although all farmers were paid more than their lands were worth, the surplus generated by the transfer was enormous and the city got most of it. He also shows, confirming Kahrl (1982) claims, that on average, farmers that sold later received a higher price. Our model can account for this feature of the data. Our focus is not on the interaction of the city *vis-à-vis* the farmers as a whole, but rather, the interaction among farmers, *given* the behavior of the city.

Our paper contributes to literature studying the privatization of public services and public procurement. There are at least two accounts of government privatization decisions. One view, which focuses on transaction costs (Williamson, 1985; Hart et al., 1997) and an alternative view (Boycko et al., 1996) emphasizing the private benefits to politicians of keeping service provision inside the government. We study a unique case, where a public actor, a city, centralizes the provision of a public good, because of strategic considerations (water provision security). We contribute to literature studying the impact of water infrastructure in urban areas. As urban environments grow, low levels of piped water usage might threaten the welfare of unconnected households posing negative externalities on neighbors (Ashraf et al., 2017). Indeed, there is plenty of evidence that large investments in water systems led to significant welfare gains in the US (Cutler and Miller, 2005) achieving near miraculous results, drastically improving life expectancy (Ferrie and Troesken, 2008) and declines in infant mortality (Alsan and Goldin, forthcoming). These results also hold for the city of Paris (Kesztenbaum and Rosenthal, 2017). This literature directly measures the impacts of having access to water in urban environments. We analyze the arguably most important water infrastructure project in the history of modern America, focusing on how the water was acquired to secure water access to urban dwellers.

There is also a rich literature in development economics studying the health impact of access to water in rural environments (Ashraf et al., 2017; Kremer et al., 2011; Shanti et al., 2010; Devoto et al., 2012). Merrick (1985) and Galiani et al. (2005) find that the access to piped water infrastructure reduces the presence of diseases.

In terms of modeling and methodology, our paper contributes to the literature in industrial organization. Takahashi (2015) studies a WoA with symmetric players and without externalities. The classical article on offshore oil drilling by Hendricks and Porter (1996) consider a simpler WoA with information externalities. More recently, Hodgson (2018) studies oil drilling in the North Sea implicitly assuming a framework like ours but solves for an equilibrium restricting the behavior of firms. Our results extend to information externalities in R&D as in Bolton and Harris (1999).

2 Background and Data

2.1 Historical Background

By 1900 the officials of the city of LA realized that the water provided by the Los Angeles river would not be enough to meet the city's future water demand, given the projected population growth. Local leaders and business owners were interested in finding an external water supply to guarantee the city growth, and to compete with San Francisco for the main economic hub on California. The solution they devised was to bring the water from the Owens River, 300 miles north of LA, to the city. For this purpose, they would need to build a large aqueduct, many dams and reservoirs and, more importantly, buy the water rights from their owners, the farmers at Owen's Valley. The value of the water would be worth much more once it arrived in the city than in the valley. To keep these rents, the city officials devised a plan to get "enough" water rights from the farmers, before the project was made public. Because the water rights were tied to the land, the city had to buy the land to get the water rights. In 1904-1905, former mayor of LA Fred Eaton traveled through the valley buying options on the purchases of the land. At this stage, the intentions of the city were not public, and farmers sold their land at "normal" prices, that is, the value of the land plus the value of water, if the water was used for irrigation in the valley.

At the time, the Federal Reclamation Service (FRS) was considering a reclamation project for the Owens Valley. The chief of the FRS in California was J.B. Lippincott, resident of Los Angeles and friend of Fred Eaton. The controversy began here. Eaton was later accused of using his association with Lippincott to imply that the options would go to the reclamation project, not to the city. Although both men denied the accusations, many farmers claim that they would have asked for a higher price, had they known the land was not going to the FRS. Fred Eaton returned to the city with all the options needed, and the plan was announced in the local newspapers. A $1.5 million bond issue is approved by the voters for a wide margin, to finance a feasibility study and to purchase the land from Eaton's options. William Mulholland is then appointed Chief Engineer of the project and in 1907 another bond is put to the voters, for $23 million, to finance the construction of the aqueduct. The aqueduct was completed in 1913. The policy in the city at the time prohibited to sell water for uses outside its limits. This meant that the nearby towns, which were also growing fast, had no option if they wanted to continue growing, but to apply for annexation to LA. The area of Los Angeles grew from 115 to 442 square miles in the following two decades.

The options bought by Eaton in 1905 were just the beginning. The city's actual growth surpassed all projections, and soon the city had to buy more land and water rights from the Owens Valley. After the project was announced, and the aqueduct built, the farmers in the valley knew that the water would be used in LA, and demanded a higher price for their plots. In the beginning, residual rights on the water were enough to satisfy the City's demand. Due to the increase in population, a new bond was passed in 1922 for $5 million. The drought of 1923 made the city to want to buy more water rights, and in 1924 two more bonds passed for $8 million each. Due to the controversy of the massive land purchase, the city is forced to buy the land and buildings on the towns within the valley, at pre-Great Depression prices. "In May [1925], the legislature passed a bill specifically allowing the payment of reparations from damages caused by the loss of water but not for those arising from the construction or operation of the aqueduct." (Kahrl, 1982, p. 296). In 1930, a new bond was issued for $38.8 million, to acquire the town properties and to buy some land in the Mono Basin.[1] Notice that, these purchases made the bulk of the total

[1] According to (Kahrl, 1982), the city paid a total of $5,798,780 for the town properties: Bishop $2,975,833; big Pine $722,635; Independence $730,306; Laws $102,446; and Lone Pine $1,217,560.

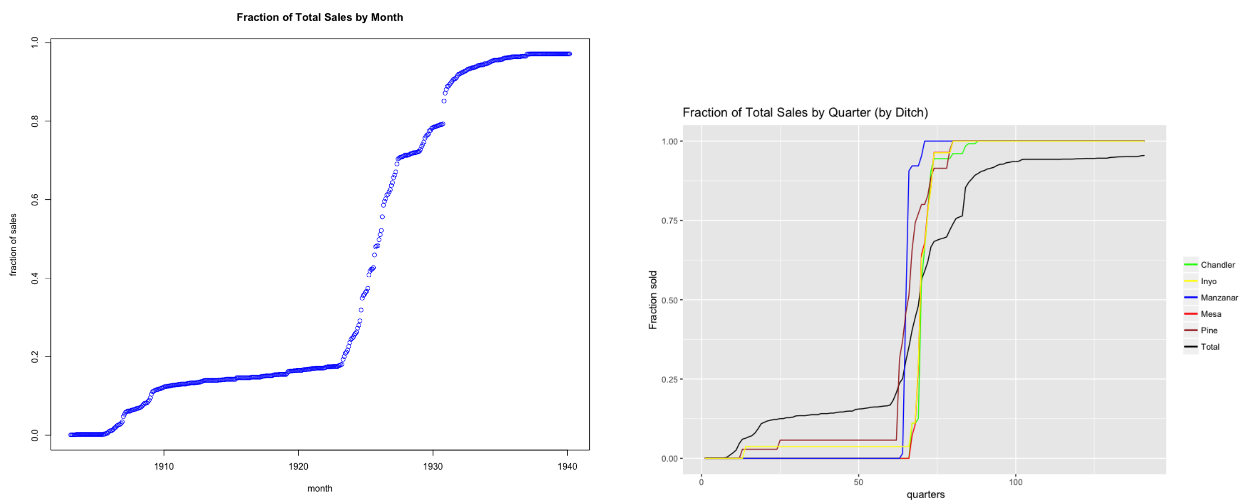
expenses, although they contained no water rights. Subsequent bonds votes to buy more water rights happened in the following decades, and by 1934 the city owns virtually all water rights in the valley, and over 95% of the farmland, and 85% of the town properties.

Within each bond, the same situation would arise. The city would have a fixed amount of money to buy land. The city would announce a committee that would evaluate the potential properties to purchase, and will make offers to each of the farmers individually. The farmers would then engage in a "war of attrition" among themselves. They knew that if they hold up, the city would offer them more money for their lands. However, when one a farmer sold their land, this would create an externality on the other farmers. After each purchase, the city would have fewer funds to continue buying up lands and will have less need for water.

Moreover, for neighboring farmers, this externality would be more significant. Farmers could get "isolated" from the river if the city bought all their neighbors' lands. If the city buys most (usually two thirds) of the farms in each ditch, it could then dissolve the ditch association and the remaining farmer would get no access to water. In this article, we focus on this game between the farmers. These externalities were important and were recognized by all parties involved. Therefore, the farmers tried initially to negotiate as one, so that they would internalize the externalities and would get a better price. They formed the Owens Valley Irrigation District (OVID) in 1923. The city then bought out the main members of the OVID. They began by buying the lands "of the oldest canal on the river [McNally Ditch] before its property owners joined the irrigation district" (Hoffman, 1981, p. 179). "Farmers illegally diverted McNally Ditch water into their own ditches, leaving the city in the position of owning a ditch without water. In retaliation, Los Angeles adopted a policy of indiscriminate land and water purchases in the Bishop area, infuriating valley people, who accused the city of 'checkerboarding."' (Hoffman, 1981, p. 179). The city then began to buy "[...] into the Owens River, Bishop Creek, and Big Pine canal companies. As with the McNally Ditch, their efforts were focused on the *key properties which controlled the points of access to the river*, so that the less favorably situated ranchers inland could be cut off from their water supply if they refused to negotiate." (Kahrl, 1982, p. 279, emphasis added)(Kahrl, 1982, p. 279, emphasis added). The city, by not buying the whole valley, presumably engaged in "a strategy of *division and attrition*---[which] was especially cruel, [...] because it placed an even larger burden of responsibility on the *farmers and ranchers who held out*" (Reisner, 1987, p. 93, emphasis added). The remaining farmers then created three smaller cartels. Each pool was a subset of the farmers owning water rights in the three major ditches. In 1927, following the collapse of the Watterson Brother's Bank, the Cashbought and the Watterson pool collapsed.

Figure 1: Sales over time.

A. Fraction of Total Sales by Month. B. Fraction of Total Sales by Ditch.



Notes: Panel A: Fraction of total sales in the data with monthly frequency. Panel B: Fraction of total sales in each ditch with quarterly frequency.

Although the city ended up buying all the land in the valley, when they were negotiating with the farmers, the farmers were unsure about how far they could sustain a hold-up. The city "made indiscriminate property purchases, leaving farmers uncertain of their neighbors' intentions." (Hoffman, 1981, p. 180). This motivates our modeling as a game of perfect information because farmers knew each other very closely and each other plot valuations, and the uncertainty was due to what the farmers would do given the city's offer. Until the 1930s, there was uncertainty as to how much land and water the city of LA was going to buy and need. This uncertainty was driven by the recurrence of droughts and by the increase in population in the city of LA. The ability to purchase land was subject to the availability of funds that came through sub sequential bonds. When the city run out of money, it was unclear whether they were going to be able to issue a new bond.

2.2 Water Law in the West

We now briefly discuss water rights in the Owens Valley. As mentioned in Libecap (2007) in Owens Valley, farmers held both appropriative and riparian surface water rights. Whereas appropriative rights can be separated from the land and be sold, riparian rights are inherent to the land and cannot be separated and sold apart. Appropriative rights are based on first appropriation, they typically are denominated in miner's inches, or as a percentage of all the water in each ditch.[2] In the data, when the farmer owns appropriative rights, we do observe whether the rights are senior or junior, and we have a measure in miner's inches or in a percentage of the flow in the ditch, which can be transformed into a measure of capacity. We compute the amount of water acre for each plot. When rights are riparian, they are attached to the land, and city typically buys the whole land surrounding the ditch, *e.g.*, "all rights in Sanger Ditch" or "all rights in Baker Creek." In those cases, we do not have a measure of water rights, because there is no explicit mention of the water capacity.

2.3 Sales Data

We created our main dataset from the transaction cards (deeds) stored at the Los Angeles Department of Water and Power (LADWP) archive in Bishop, Inyo County. In Figure 2. A we can see a sample in that transaction card. Each transaction card refers to a particular Section, in a particular Township and a particular Range, all of them in Mount Diablo Meridian (M.D.M.). Typically, one section corresponds to a square of one-mile times one-mile, or 640 acres. Thus, a quarter of a section corresponds to 160 acres; a quarter of a quarter corresponds to 40 acres; and half of a quarter of a quarter corresponds to 20 acres, as in Figure 2.A. This particular example is an easy one, but in some cases the same farmer owned several plots, sometimes non-contiguous. In many cases, the plots were not rectangular, and the geo-codification is more cumbersome. However, we were able to geo-code all the plots. This is important not just to be able to create the maps, but also to create variables that are relevant to our analysis, as we explain below in subsection 2.4. For our baseline analysis, we merge all continuous plots owned by the same farmer, and treated the merged plot as a single plot.

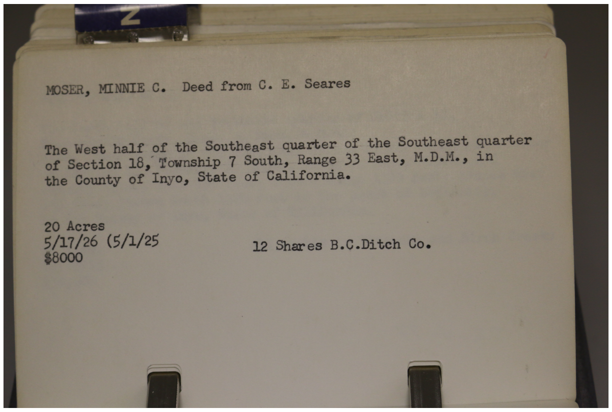
Table 1 shows summary statistics for the variables used in the analysis. As shown in Figure 2.A, we have not only the year of purchase but the exact date of purchase.[3] In the main analysis, we only consider transactions between 1905 and 1935. The reason is that, as explained above in subsection 2.1, before 1905 farmers were unaware of the intentions of the city, and they sold their land to Fred Eaton. By 1935, the city owned all the water rights and virtually all the non-federal lands in the Owens Valley. There are some sporadic transactions in the 1970s and 1980s, but they are very different in nature to the land purchases of the beginning of the 20^{th} century.

[2]They could be senior or junior rights. During a dry season, all the senior rights have to be fulfilled, before any junior rights claimant get any water.

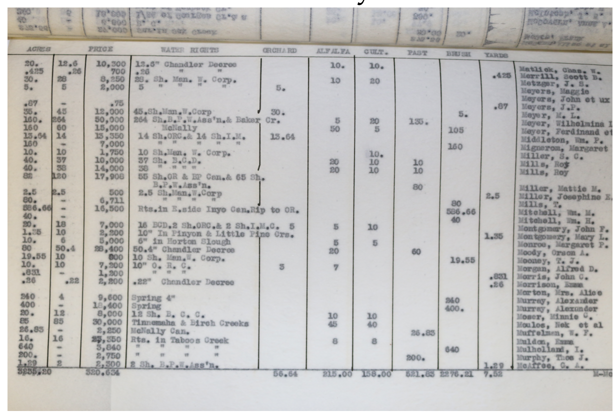
[3]For many of the cards, we do see two dates. We know that the later date, or the only date when there is only one, was the date when the land was sold. We believe that the first date is the date when the offer was made.

Figure 2: Sample Pictures from data collection.

A. Transaction Card.



B. Survey.



Notes: Panel A: Caption of a transaction between Minnie C. Moser and the city of Los Angeles. Panel B: Caption of the survey conducted by the city of Los Angeles, where the plot owned by Minnie C. Moser is seventh from the bottom.

In addition to the date of purchase, we have information regarding the size of the land and the amount paid for it, which we obtain directly from the cards. The cards do contain information regarding water rights, but in a format that is not directly comparable across farmers. In some cards, the information is regarding the number of shares, sometimes it says a percentage of all rights in a particular creek, and sometimes it mentions first or second rights using miner's inches. All those measures are homogeneous and comparable within a ditch, but not across ditches. To get a comparable measure of water rights across all farmers, we merged our dataset with the data collected by Gary Libecap. Gary Libecap's work cited above is based on the data available at the LADWP archive in Los Angeles. We merge our data with his data to obtain a uniform measure of water rights.[4]

In addition to the transaction cards, we complemented the data with the surveys conducted by the surveyors hired by LADWP. Figure 2.B, shows a sample picture of the summary of the survey. We merge the dataset created using the transaction cards with the survey data using the names of the farmers. In the survey, we can also see how not only the name but also the acreage and the water rights data also match with the information

[4]In Libecap's dataset, there is a measure of annual water acres for each farmer. Hence, for the farmers in his dataset, we have an exact measure of water acres. For reasons that are not clear to us, his dataset contains fewer farmers than ours. Whereas we were able to find merge all farmers in his dataset in our dataset, there are about 600 farmers in our dataset that do not appear in his dataset. However, most of those farmers have water rights in the same ditch as another farmer that appears in Libecap's dataset. We assume that all shares and all miner inches in a given ditch convey the same number of water acres, and we use Libecap's data to extrapolate the water acres for those farmers.

Table 1: Summary Statistics.

Variable	Mean	SD	Min	Max	Obs
Year	1,927	13.4	1,903	1,997	1,390
Acres	209.6	741.9	1	11,918	1,390
Price	26169	104594	1	2,000,000	1,250
Water Acres	257.3	882.45	0	17,850	1,381
Distance to the river	5,128	9,987.184	0	250,957	1,390
Distance to Mono lake	111,920	44,454.43	0	434,895	1,390
Distance to Owens lake	69,446	41,558.31	0	246,874	1,390

Notes: Summary statistics for selected variables. *Year* is a numeric variable that measures the year where the plot was sold. *Acres* is the number of acres of the property sold. *Price* is the final price that the farmer received for her plot. The lower number of observations with prices is due to some farmers exchanging their land for another piece of land owned by the city. *Water Acres* represents the amount of water rights, measured in water acres per year, that each farmer owns.

in the cards. The survey, however, has an extra piece of information not present in the transaction cards, but that is an essential determinant of the price paid: land use. In the survey, the land for each farmer is decomposed on how many acres are used for each of the following six categories: Orchard, Alfalfa, Cultivated, Pasture, Brush and Yards.

Based on the data used, we can distinguish three generations of work on the topic, and the same categorization applies to other settings. The first generation includes historians such as Hoffman (1981) and Kahrl (1982), who used summary data to draw their conclusions. The sources that they cite most commonly are the summaries written by Thomas H. Means for the city of LA. Given the lack of detailed or individual data they could conclude the total amount that the city spent, but not definite conclusions regarding the evolution of prices, the size of the surplus or the distribution of such surplus. The second generation includes the work by Gary Libecap (Libecap, 2005, 2007, 2009). He used individual data, but does not have the geographical location of the plots. He documented the upward trajectory of prices paid to the farmers and have robust measures of the surplus and concluded that the city got most of the surplus, and the farmers that sold later got a better price on average. The lack of geographical information also limits the analysis because there are some omitted variables that affect the value of the plot. This paper is a third generation on the topic and, in addition to detailed individual data, it uses geo-coded variables that help explain the heterogeneity of the plots. The crucial innovation, however, is that geographical information allows us to test and compute spatial externalities.

This allows us to address whether the city was in effect "checkerboarding" farmers and whether it was efficient or not.

2.4 Geo-location data

As mentioned above, the transaction cards provide a detailed description of the exact location of each plot. We geo-coded 2,750 plots. Figure 3.A shows the land holdings from the main sellers, *i.e.*, those who received over $1 million for their land.[5] Notice that the State of California was by far the largest seller. Fred Eaton appears as the second largest seller, despite not being a farmer or a landowner in the valley before 1905. He acted as an intermediary who bought land from the farmers and sold it to the city. Most of the land is in the lower part of the valley, close to Owens Lake. However, it is worth noticing the large plot of land sold by Eaton in Mono County. This plot of land correspond to the Rickey ranch, covering 11,190 acres and purchased for $425,000 after "a week of Italian work" by Eaton (cited by Reisner, 1987).[6] The ranch had the best natural spot for a reservoir (Long Valley), and its contentious sale to the city by Eaton destroyed the friendship between him and Mulholland.

In addition to creating the maps, which are very useful to have a better understanding of the data, the goal of the geo-location is to create more variables. For each polygon geo-coded, we can merge it with data available in GIS (Geographical Information Systems). After the merge, we have important variables such as altitude, roughness, slope, suitability and distance to the Owens River. All of which are important determinants of the quality of the land and thus the price received. We are especially interested in the distance to the river because, based on the historical literature, we conjecture that within a ditch, farmers whose plots are closer to Owens River, would create a larger externality in other farmers, than farmers that are further away. Finally, geo-coding the plots for all farmers allow us to compute distances between farmers, and to perform a rigorous spatial analysis. The distance between the farmers' plots would also affect the size of the externality created by other farmers' sales.

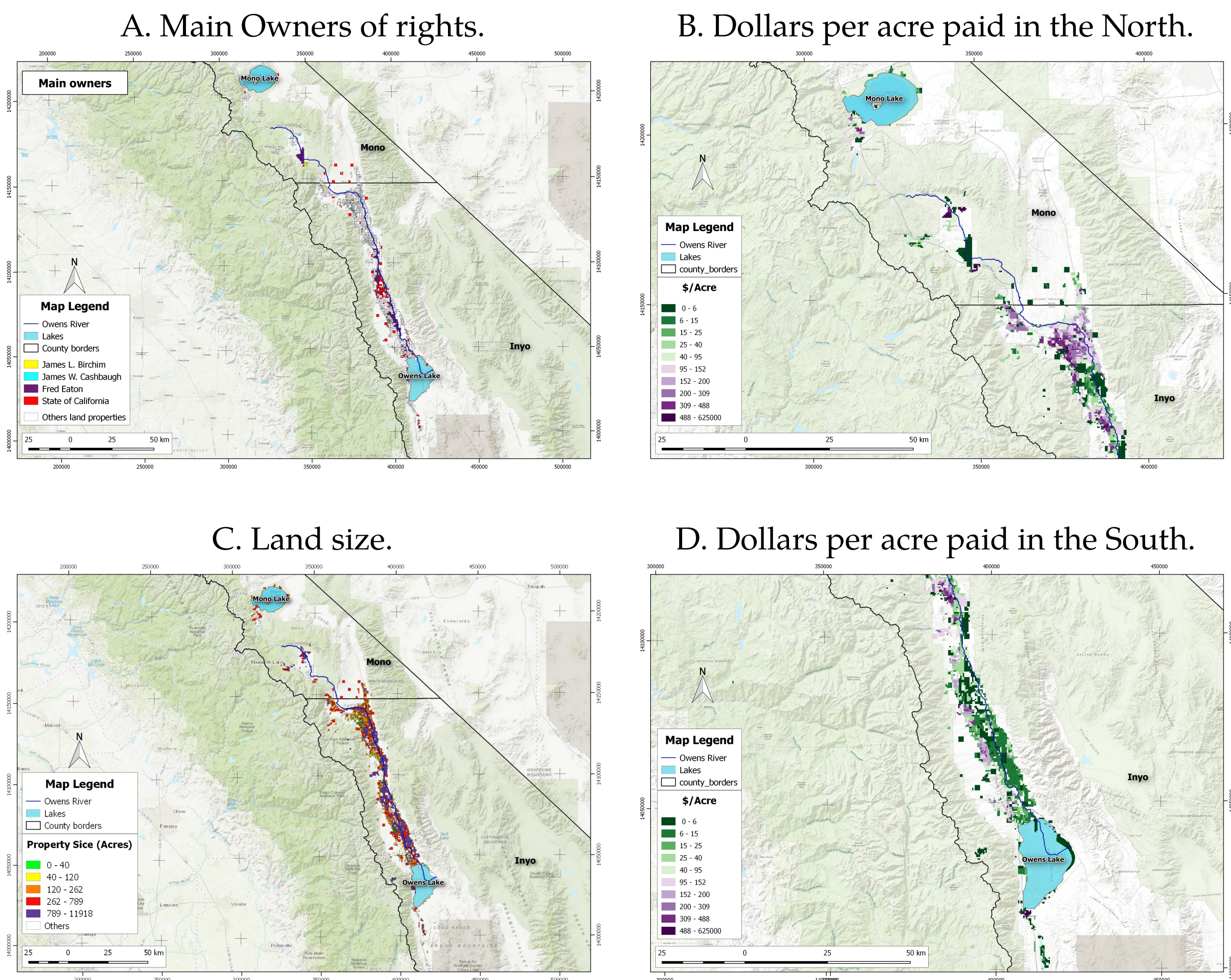
3 Preliminary Evidence

In this section, we present descriptive statistics and reduced-form analysis that, together with the historical evidence presented in Subsection 2.1 motivates the design of

[5]James Birchim received $2 million in 1981 for 646.12 acres. James Cashbaugh received $1.4 million in 1985 for 636.66. Because these sales were so late, they are not included in our analysis.

[6](Reisner, 1987) also implies that Lippincott favored the application of the Nevada Power Mining and Milling Company, founded by Thomas B. Rickey, over that of the Owens River company, for the building of a power plant in the valley. Lippincott recommendation was then key to convince Rickey to sell the ranch.

Figure 3: Digitized maps in Owens Valley.



Notes: Panel A: Map with the main water rights owners, *i.e.*, those who received over $1 million. Notice that Fred Eaton is listed although he was an intermediary. Panel B: Map of the dollars per acre paid for each plot in the north of the Owens Valley. Panel C: Map of the total area holding of each seller. Panel D: Map of the dollars per acre paid for each plot in the south of the Owens Valley.

our theoretical modeling and the structural estimation. We first run a Moran test of spacial correlation. Table 2 shows that there is a spatial correlation on the date of sale within the whole valley, which suggest spatial externalities and highlight the importance of the geo-coded data. We also compute the unconditional hazard rates of selling times by ditch. Figure 4 shows that the shape of the hazard rate of selling times is not constant over time. That means that a regular regular linear regression would not capture the variation in the data. Finally, we show several reduced-form regressions to explain whether a farmer sold or not and at what price, in a dynamic setting. First, we estimate a Cox model, which can capture both time-variation and farmer specific variation. Table 5 show the results from the Cox regression. We also run a linear regression including variables that relate to spacial externalities. Table 6 shows that there are externalities that affect individuals over time differently. Therefore, a Cox model should not be used, because it cannot capture this individual-time variation, *i.e.*, a Cox model requires the no individual-time variation. The individual-time variation shown in Table 6 is, however, constant between two consecutive exits, *i.e.*, constant within a sub-game. This means that we should model the behavior or the farmers as an exit games with time-varying hazard rate that resets every time a farmer in the same ditch sells. All of the above motivate our choice of a War of Attrition model with externalities and time-varying valuations.[7]

3.1 Spatial Correlation

In Table 2, we present the results of a Moran's I test of spacial correlation. The Moran's I is a non-parametric test that measures the spacial correlation of a particular value. To gain intuition let's consider an example where half or the farmers sell at one time and the other half sell at another time. If all who sold first and clustered together in one side, and all who sold later are clustered together on another side, then the Moran I statistic would be equal to 1. If, on the contrary, the farmers who sold first are not neighboring any other farmer who sold first, as in a checkerboard, then the Moran I statistic would be equal to -1. Finally, if farmers sold their land at random times, the Moran I statistic would be equal to 0.

The first row in Table 2 shows the results of a Moran I test where the variable of interest is Price per Acre. The test shows no spacial correlation on Price per Acre for the whole sample. This would seem to contradict the hypothesis proposed by Kahrl (1982) that farmers that are "clustered" along the river would get higher prices per acre for their plots. Notice, however, that this is a strong weak test, because it considers linear relations, and

[7] In Appendix **??**, we show robustness results that confirm that the patterns are due to the externalities imposed by the sale of a neighbor, and not other hypotheses.

it imposes the same relation across the whole map. Kahrl's hypothesis a strong spacial relation between those right at the river, and those far from the river, and presumable only for those in the main ditches, but a zero relation between any other pair of plots. Therefore, the zero spatial correlation for the whole map does not necessarily contradicts Kahrl (1982).

The second ow in Table 2 shows the results of a Moran I test where the variable of interest if the Date of Sale. It shows a strong and significant positive spacial correlation in dates of sale. This is consistent with our motivation of spacial externalities across neighboring farmers. In other words, when a farmer sells her plot, her neighbors are more likely to sell. Notice, that this correlation could also be spurious. Given the nature of the city's land purchases, the city historically begun making offers to farmers in the south of the Valley, and as its water demands increased, it made offers to farmers more and more to the North. In other to control for that, we have decomposed the sample into several time windows and run the test within those time windows. The plots before 1906 correspond to the purchased made by Fred Eaton, without the farmers knowing that they were selling to the city. The sales from 1907-1912 correspond to sales after the city announce the construction of the aqueduct, but before it was finished, we see a positive spacial correlation here too. There are a few sales after the aqueduct was built, but before the city passed a new bond to buy more water (1913-1921). The lack of spacial correlation here reflects the fact that those were 8 reluctant farmers that sold at random times, for idiosyncratic reasons. In 1922 the city decided to raise more funds to buy more water. This is our period of interest, not only because it contains most of the sales of water and land, but also because it is where the conflict was more evident. We divided the sample here into two-year windows and we can see that the spacial correlation is very high and significant. This implies that the high spacial correlation that we see in the second row is not an artifact of the city buying first the southern part of the valley, and later the northern part, but rather strong spacial correlation across farmers with lands *within* the same part of the valley.

3.2 Hazard Rates

Figure 4 shows the unconditional hazard rates of farmers exiting by quarter. We compute the hazard rates following a similar approach as Hendricks and Porter (1996). We restrict attention to farmers that belong to a ditch with more than five farmers. The first period for each ditch is set to be the quarter where the first farmer in such ditch sold her land to the city. We can see that during the first 30 quarters, the hazard rate is erratic, but with a downward trend. As we show later in Section 4, a downward hazard rate is consistent

Table 2: Spacial Correlation.

Moran's I Test Results

Model	Observations	Moran I statistic	p-value
Sample P/A	1158	-1.21E-03	0.5139
Sample Timming	1158	0.333178241	2.20E-16
Timming (<1906)	38	0.38125082	0.0004194
Timming (1907-1912)	108	0.161978265	0.02568
Timming (1913-1921)	8	-0.12837051	0.4805
Timming (1922-1923)	90	0.265955734	0.000847
Timming (1924-1925)	135	0.278952641	4.11E-05
Timming (1926-1927)	154	0.504066624	1.42E-12
Timming (1927-1928)	188	0.062588665	0.1414
Timming (1928-1929)	178	0.367887174	2.16E-08
Timming (1930>)	259	0.503678295	2.20E-16

Notes: Results from a Moran I Test of Spatial correlation on the year that each farmer sold their plot. *Sample P/A* corresponds to the spatial correlation of price per acre. *Sample Trimming* corresponds to the spatial correlation of year of sale, taking all the observations between 1906 and 1935. *Timing* (X) corresponds to the spatial correlation of year of sale, taking all the observations included in X.

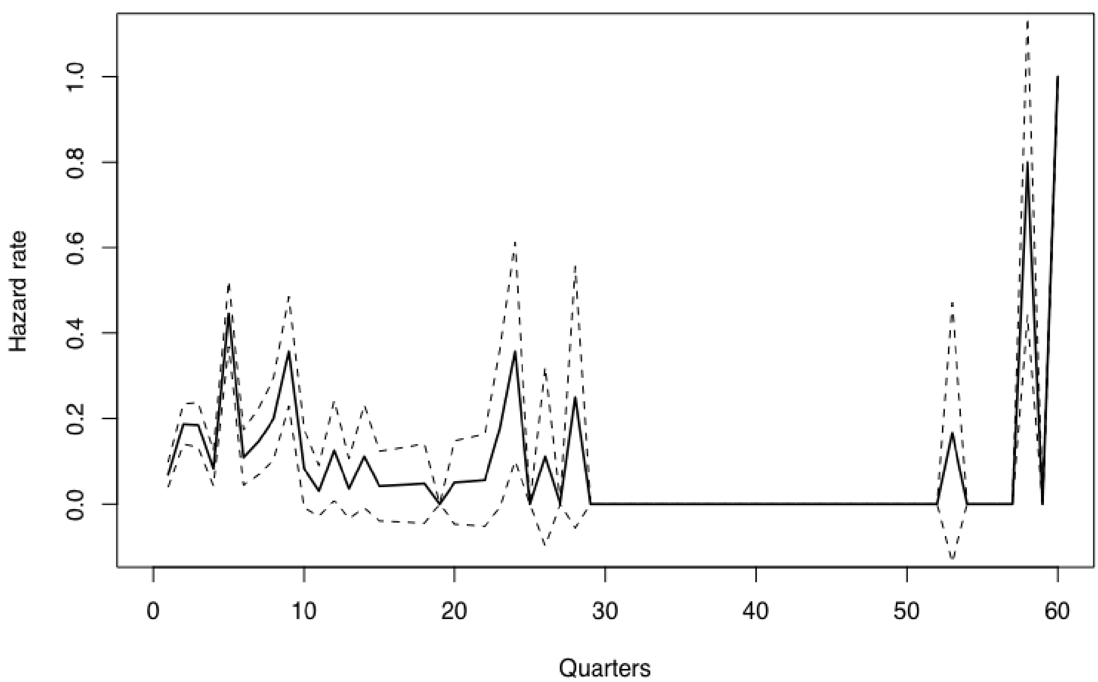
with an upward value of waiting. After 30 quarters, most of the farmers have sold, and a handful of outliers explain the postive hazard rate after 50 periods.

We can see that there are periods where the hazard rate is high, and periods when it is low. This suggest that the baseline hazard rate is not constant and that we should incorporate dynamics in our estimation. Moreover, not only is the hazard rate different for different periods, it also displays many changes in the sign of the slope, going from very high to very low values. This is consistent with our interpretation of spacial externalities. After a key farmer sells, it produces a negative externality on the remaining farmers, who rush to sell. Then things are quiet for a while until another key farmer, or an accumulation of small farmers, triggers another run.

3.3 Reduced Form Analysis

In Table 3 we regress several covariates on the price paid by the city of LA. We include variables that we obtain from the records in the archive that would affect the price such as Acres and Water Acres. The results are not surprising, and the sign and size of the effects are reasonable. In addition to these variables, we linked each individual plot with climatic and geophysical information. The climatic and geophysical variables affect productivity and are the usual inputs to compute land productivity. The additional climatic variables are Annual precipitation, Annual snow and Annual humidity. The additional geophysical

Figure 4: Unconditional Hazard Rates.



Notes: Unconditional hazard rate for farmers that belong to a ditch with 5 or more farmers.

variables are Mean elevation, Mean slope and Mean Roughness. Below we use all these variables as controls when estimating the effects of externalities.

We included climatic and geophysical variables to address the concerns that the plots of land were heterogeneous on their productivity and that these unobserved differences in productivity were driving the results. We do find that there are indeed differences in climate and physical characteristics in the valley. Moreover, as the results in Table 3 show, these differences affected the price that farmers got, *i.e.*, farmers with more productive lands received a higher price per acre for their land.

Table 4 use the same variables as Table 3 above, but the explanatory variable is the Date of Sale, *i.e.*, days since January 1, 1900. The negative coefficient on the variable Acres means that farmers who owned larger plots, typically big landowners or big ranchers sold earlier than smaller farmers. The large negative and significant coefficient on Mean roughness means that farmers with poor quality land also sold earlier.

The econometric specification in Table 4 implicitly assumes that the probability of selling is constant over time. Based on the estimated hazard rate in Figure 4, the hazard rate is not constant. Hence, we also estimate a Cox model. Table 5 shows the results of the Cox model. Notice that the negative coefficient on the variable Acres means that farmers who owned larger plots have a higher probability of selling and, thus, sold earlier than smaller farmers. Overall, the coefficients are similar, but with opposite signs, than in Table 4.

The advantage of the Cox model is that it does not restrict how the probability of selling changes over time. The Cox model is a particular case of Proportional Hazard Rate Models (PHRM). In a PHRM the hazard rate, or the individual instantaneous probability of selling, can be decomposed into two multiplicative terms: one term (baseline hazard) that depends only on time and one term that depends only on individual characteristics. Therefore, the Cox model does not impose any restriction on how the probability of selling changes over time, except that the baseline hazard is the same for all farmers beginning at time zero, regardless of the actions of any farmer. We are interested on estimating spacial externalities, *i.e.*, the effect that a farmer's sale have on neighboring farmers. However, the Cox model is not flexible enough to estimate such model. To illustrate this point and to show evidence of spacial externalities we extended the cross sectional dataset and transform it on a panel dataset. In the panel dataset, we created variables that relate to the actions take by other farmers in the same ditch as a given farmer.[8] The variables are: Distance to Owens river (m), Percentage of Sales to date (percentage of farmers on the same ditch that have sold), Percentage of Shares to date (percentage of water rights in the ditch

[8] In the appendix, we also show similar results for neighboring farmers, defined as the four farmers with the closest plots to a given farmer.

Table 3: Price Determinants.

	Dependent variable:			
	Price per acre			
	(1)	(2)	(3)	(4)
Acres	−0.06	−0.06	−0.05	−0.04
	(0.08)	(0.08)	(0.08)	(0.08)
Water acres		−0.07	−0.15	−0.15
		(0.28)	(0.28)	(0.28)
Annual precip.			241.05	201.36
			(1,028.14)	(1,032.13)
Annual snow			−102.30	−88.16
			(381.30)	(383.05)
Annual humidity			271.05	242.92
			(857.80)	(862.52)
Mean elev.				−0.73
				(0.84)
Mean slope				0.002**
				(0.001)
Mean roughness				−682.68**
				(321.48)
Constant	498.16***	511.14***	−11,158.75	−8,514.23
	(119.79)	(128.95)	(38,060.72)	(38,265.80)
Observations	972	971	971	969
R^2	0.001	0.001	0.004	0.01
Note:		*p<0.1; **p<0.05; ***p<0.01		

Notes:

Table 4: Time of Sale Determinants.

	Dependent variable:			
	Days between 01/01/1900 and sale			
	(1)	(2)	(3)	(4)
Acres	-0.53^{***}	-0.53^{***}	-0.48^{***}	-0.50^{***}
	(0.05)	(0.05)	(0.05)	(0.05)
Water acres		0.34^{*}	0.19	0.26
		(0.19)	(0.19)	(0.18)
Annual precip.			-893.81	-980.76
			(677.80)	(660.63)
Annual snow			344.38	345.66
			(251.37)	(245.18)
Annual humidity			-723.63	-714.02
			(565.50)	(552.07)
Mean elev.				3.06^{***}
				(0.52)
Mean slope				0.001^{**}
				(0.001)
Mean roughness				-378.33^{*}
				(203.83)
Constant	$9{,}192.13^{***}$	$9{,}141.71^{***}$	40,707.15	37,404.84
	(80.63)	(86.39)	(25,091.66)	(24,492.89)
Observations	990	988	988	986
R^2	0.09	0.10	0.15	0.20
Note:		*p<0.1; **p<0.05; ***p<0.01		

Notes:

Table 5: Cox Regressions.

	Dependent variable:			
	Days between 01/01/1900 and sale			
	(1)	(2)	(3)	(4)
Acres	0.0001***	0.0001***	0.0001***	0.0002***
	(0.00002)	(0.00002)	(0.00002)	(0.00002)
Water acres		0.0002***	0.0002***	0.0001**
		(0.0001)	(0.0001)	(0.0001)
Annual precip.			−0.023***	0.257
			(0.005)	(0.297)
Annual snow				−0.087
				(0.110)
Annual humidity				0.194
				(0.248)
Mean elev.				−0.002***
				(0.0002)
Mean slope				−0.00000***
				(0.00000)
Mean roughness				0.245**
				(0.096)
Observations	990	988	988	986
R^2	0.016	0.023	0.042	0.146
Note:		*p<0.1; **p<0.05; ***p<0.01		

Notes:

that have been sold), SD-Percentage shares to date (the standard deviation of the percentage of water rights in the ditch that have been sold), Mean-price/acre to date (the average across farmers on the same ditch on the price per acre received for their plots), SD-price-acre to date (the standard deviation across farmers on the same ditch on the price per acre received for their plots), Percentage acres to date (percentage of land rights in the ditch that have been sold) and SD-Percentage acres to date (the standard deviation of the percentage of land rights in the ditch that have been sold).

Table 6 shows the results of running a Logit regression on the panel dataset on the set of controls that determine the price and one externality variable at a time. Notice how Distance to the Owens River does not seem to have an effect on whether a farmer sold or not. The variables representing the mean characteristics (their number, their water rights and their land rights) of the farmers who sold, are all positive and significant. This is consistent with positive spatial externalities, and those externalities being larger for farmers with larger plots and more water rights, *i.e.*, key farmers. Moreover, the variables that measure the standard deviation of those characteristics are also positive and significant, which suggest convex effects, *i.e.*, larger farmer have more than proportional effects on their neighbors upon selling. In summary, the results in Table 6 are consistent with the presence of spacial externalities. The econometric specification in Table 6, however, implicitly assumes that the hazard rates are constant, in between two farmer's exits.

The evidence presented in this section could be summarized as follows: the hazard rates of selling are not constant over time, thus, a linear regression is not adequate; the probability of selling over time depends on the action by other farmers (spacial externalities), thus a Cox model is not flexible enough. Therefore, we need an econometric model that allows for spacial (farmer to farmer) externalities and allows the probabilities of selling to change over time, in between farmers sales. In the next section, we present such a model and solve for its equilibrium.

4 Econometric Model

This section introduces the theoretical model. We model the game between the farmers as a game of perfect information, unlike (Takahashi, 2015), which estimates a model of imperfect information. Using the arguments in Harsanyi (1973), as we explain in more detail below in subsection 4.4, one can see that the two games are observationally equivalent. In other words, the data can be rationalized either by a game of perfect information, or by a game of imperfect information.[9] We choose to model the interaction as a game

[9]Typically the game of imperfect information has a unique equilibrium, while the game of perfect infor-

Table 6: Externalities Effects.

	Dependent variable:							
	Sale in quarter							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dist. to Owens River (m)	−0.00002							
	(0.00003)							
Prct. sales to date		7.153***						
		(0.235)						
Prct. shares to date			6.254***					
			(0.187)					
SD-Prct. shares to date				18.242***				
				(1.070)				
Mean-price/acre to date					0.005***			
					(0.0001)			
SD-price/acre to date-						0.003***		
						(0.0001)		
Prct. acres to date							6.807***	
							(0.197)	
SD-Prct. acres to date								−1.939
								(1.874)
Constant	−4.152***	−5.908***	−2.077	−5.398***	−0.056	−1.450	−4.416***	1.555
	(1.165)	(1.546)	(1.778)	(1.078)	(1.973)	(1.638)	(1.487)	(1.648)
Controls?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	23,827	23,827	23,827	23,827	23,827	23,827	23,827	997
Log Likelihood	−1,570.760	−745.600	−822.786	−1,437.158	−1,078.121	−1,211.416	−822.166	−563.115
Akaike Inf. Crit.	3,159.519	1,509.200	1,663.572	2,892.315	2,174.242	2,440.832	1,662.331	1,144.230

Note: *p<0.1; **p<0.05; ***p<0.01

Notes:

of perfect information because we think that it is realistic in the empirical setting studied here. The historical literature has pointed out how all farmers were informed both about the actions of other farmers selling their plots to the city, the amount they got offered and the characteristics of each plot.[10]

4.1 Theoretical Model

We model the interaction between the farmers as a War of Attrition (WoA) based on the results in Catepillan and Espín-Sánchez (2019), when they take as given the offer made by the city, and the contingent offers that the city would make over time. One can think of each game presented here as the game between farmers in the same ditch. There are N farmers with each farmer (he) indexed by $i = 1, ..., N$ and the city of LA (she) as $i = 0$. The game begins a $t = 0$ and time is continuous. There $t = 0$ the city makes an offer to each farmer. The offer consists of a price $V_{i\cdot}(0)$ that the farmer would receive for their plot if she sold at $t = 0$. There is perfect information and we assume that the city can commit to a stream of future offers to each farmer. Future offers are then common knowledge and may depend on the time since the game began, indicated by the scalar t; the number of farmers that have sold at a given point denoted by the scalar k; and in general in the identity of each of the farmers that have sold at a given point, denoted by the set $\mathcal{K}$. At each instant in time, a farmer decides whether to stay in the market or to sell his farm to the city. While staying, each farmer pays a monetary unitary instantaneous cost. The interpretation of this instantaneous cost in continuous time is that of discounting.

It is important to make a distinction between the whole game, that involves all the farmers in a ditch and their selling times, and each stage game, that involves only the subset of farmers that have not sold up to that point. We can focus on each stage game, when farmers take the continuation value after another farmer sells as given. In a stage game with n remaining farmers, the value of a given farmer of selling is just the offer made by the city for that case $V_{i\mathcal{K}}(t)$. Notice that the offer depends on the time, the identity of the farmer and the set of farmers that have sold already. If a farmer does not sell, his continuation value, that is the value of being active in the next stage game, would depend on the identity of the farmer who sold. The continuation value of the farmer is then $W^j_{i\mathcal{K}}(t)$ when farmer j sold his plot at time t. Because the farmer is deciding whether to sell or not, the important element is the difference between selling at time t, which involves an

mation has many. We focus on the equilibria of the perfect information game where all farmers use mixed strategies, as this is the one that rationalized the data and the one that is observationally equivalent to the game of imperfect information.

[10] Pearce (2013)documents how close the community was in the small towns in the valley and how everyone knew even when their neighbor took the train to LA to sign the sale.

immediate reward $V_{i\mathcal{K}}(t)$, and not selling at time t, which involves a continuation value $W^j_{i\mathcal{K}}(t)$. We denote this difference by $\Delta^j_{i\mathcal{K}}(t) \equiv W^j_{i\mathcal{K}}(t) - V_{i\mathcal{K}}(t)$. Finally, we define $\Delta_{i\mathcal{K}}(t)$ as the expected difference between selling or not at time t. In particular

$$\Delta_{i\mathcal{K}}(t) \equiv \sum_{j \neq i} f_{j\mathcal{K}}(t) W^j_{i\mathcal{K}}(t) - V_{i\mathcal{K}}(t) = W_{i\mathcal{K}}(t) - V_{i\mathcal{K}}(t) \tag{1}$$

where $f_{j\mathcal{K}}(t)$ is the instantaneous probability that farmer j sells at time t. Notice that the expected difference is not a fundamental element of the game, not even a fundamental element of the stage game, because it depends on the probability of selling of each of the other farmers $f_{j\mathcal{K}}(t)$, which are equilibrium objects.

To solve the equilibrium, we make one assumption regarding the evolution of $\Delta^j_{i\mathcal{K}}(t)$ over time.

Assumption A1: The difference in valuation between selling or not for each farmer is separable in time, and all farmers have a common time component

$$\Delta^j_{i\mathcal{K}}(t) = \Delta^j_{i\mathcal{K}} \cdot v(t)$$

Assumption A1 implies that the "shape" of $\Delta^j_{i\mathcal{K}}(t)$ over time is the same for all farmers. The intuition is that although the value is different for each farmer, and is changing over time, the "shape" of the change is common to all farmers. It is worth noticing that in the classical WoA models, the value of the "prizes" that the players get does not change over time, *i.e.*, in the classical WoA $v(t) = 1$. This means that both the values of selling and the continuation values are constant over time. A constant $\Delta^j_{i\mathcal{K}}(t)$, as we show below, implies a constant probability of selling over time, which means that the distribution of selling times will have a constant hazard rate. Therefore, the assumption of constant values is equivalent to assuming that the distribution of selling times is exponential. Below we show how there is a direct relation between the shape of the valuations over time and the shape of the distribution of selling times, *i.e.*, given a function of valuations over time $v(t)$, there is a unique distribution of selling times in equilibrium and given a distribution of selling times in the data, the is a unique function of valuations over time that rationalizes it. In Subsection 4.4 we show how our data allow us to non-parametrically identify the distribution of valuations. For simplicity and we chose a flexible parametric form for the estimation.

4.2 Equilibria

We now show how to solve for the unique equilibria where all farmers are using mixed strategies.[11] As defined above the value of staying until the next stage for farmer i when farmer j sells at time t in a stage game when the set $\mathcal{K}$ of farmers have already sold is $\Delta_{i\mathcal{K}}^{j}(t) = \Delta_{i\mathcal{K}}^{j} \cdot v(t)$. Since the cost of staying is unitary, the cost function over time is $C(t) = t$. We assume that $v(t)$ is differentiable. The utility for farmer i of staying until time t, given that farmer j is leaving at time s with probability $f_{j\mathcal{K}}(s)$ is

$$U_{i\mathcal{K}}^{j}(t) \equiv \sum_{j \neq i} \int_{0}^{t} \left[\Delta_{i\mathcal{K}}^{j} \cdot v(s) - s\right] f_{j\mathcal{K}}(s) \prod_{k \neq i, k \neq j} \left[1 - F_{k}(s)\right] ds - t \left\{\prod_{j \neq i} \left[1 - F_{j\mathcal{K}}(t)\right]\right\} \quad (2)$$

That is, farmer i gets $\left[\Delta_{i\mathcal{K}}^{j} \cdot v(s) - s\right]$ if farmer j is the first to sold, and does so at time $s < t$; and farmer i gets $-t$ if nobody sells before t. The derivative of the utility exists and we get the following expression

$$\begin{aligned} \frac{dU_{i\mathcal{K}}^{j}(t)}{dt} &\equiv \sum_{j \neq i} \left[\Delta_{i\mathcal{K}}^{j} \cdot v(t) - t\right] f_{j\mathcal{K}}(t) \prod_{k \neq i, k \neq j} \left[1 - F_{k\mathcal{K}}(t)\right] - \left\{\prod_{j \neq i} \left[1 - F_{j\mathcal{K}}(t)\right]\right\} \\ &+ \quad t \sum_{j \neq i} f_{j\mathcal{K}}(t) \prod_{k \neq i, k \neq j} \left[1 - F_{k\mathcal{K}}(t)\right] \\ &= \left\{\prod_{j \neq i} \left[1 - F_{j\mathcal{K}}(t)\right]\right\} \cdot \sum_{j \neq i} \left[\Delta_{i\mathcal{K}}^{j} \cdot v(t) \cdot f_{j\mathcal{K}}(t) - 1\right] \end{aligned} \quad (3)$$

In equilibrium, the expected utility of not selling for any farmer that is using a mixed strategy need to be constant. Otherwise, the farmer would sell (if his expected utility is negative) or not sell (if his expected utility is positive). Thus, in equilibrium, $\frac{dU_{i\mathcal{K}}^{j}(t)}{dt} = 0$ and the probability that farmer j sells at time t, $f_{j\mathcal{K}}(t)$, is the strategy followed by farmer j that makes all other farmers indifferent between selling or not, that is, $\lambda_{j\mathcal{K}}(t)$. This produces the following equilibrium condition for farmer i

$$\sum_{j \neq i} \left[\Delta_{i\mathcal{K}}^{j} \cdot v(t) \cdot \lambda_{j\mathcal{K}}(t)\right] = 1, \forall i \quad (4)$$

Notice that we have one equilibrium condition for each remaining farmer, n equations in total. The system of n equations will solve the strategies for each farmer $\lambda_{i\mathcal{K}}(t)$. We do so in two steps. The first step is to solve for $\phi_{i\mathcal{K}}$, such that

$$\sum_{j \neq i} \Delta_{i\mathcal{K}}^{j} \cdot \phi_{j\mathcal{K}} = 1, \forall i$$

[11] See Catepillan and Espín-Sánchez (2019) for details and a broader discussion of equilibria.

This is a linear system of equations and it is easy to solve. Appendix B solves the case for three farmers to show the intuition behind the role of the values and the externalities on the probabilities of selling. The intuition extends to more than three farmers but the algebra is cumbersome. Then, the strategy for farmer i, that is the probability distribution of selling over time, must follow a hazard rate that satisfies equilibrium condition 4

$$\lambda_{i\mathcal{K}}(t) = \frac{1}{\phi_{i\mathcal{K}} \cdot v(t)} \tag{5}$$

Therefore, the distribution of selling times for farmer i in game $\mathcal{K}$ is

$$F_{i\mathcal{K}}(t) = 1 - c \cdot exp\left[-\int_0^t \frac{\phi_{i\mathcal{K}}}{v(s)} ds\right] \tag{6}$$

where c is the constant of integration that makes $F_{i\mathcal{K}}(t)$ a probability distribution. Notice that equation 5 is key to identify the shape of $v(t)$. In the model, using equation 6, for each $v(t)$ we can compute the exact shape of the distribution of selling times for each farmer in each game. When we look at the data, we can see the empirical distribution of selling times for each farmer for a given game, $\widehat{F}_{i\mathcal{K}}(t)$. With that distribution, we can compute the empirical hazard rate of selling times for each farmer for a given game, $\widehat{\lambda}_{i\mathcal{K}}(t)$. Then, using equation 5 we can compute the shape of $v(t)$ and estimate the externalities using the estimates on $\phi_{i\mathcal{K}}$.

4.3 Linear value function

The model presented above computes the equilibria for any value function $v(t)$. We now show the results when the value function is linear. Therefore the value function is now

$$v(t) = \alpha + \beta t \tag{7}$$

Following equation 5 and equation 7 we can write the hazard rate of selling times for farmer i as

$$\lambda_{i\mathcal{K}}(t) = \frac{1}{\phi_{i\mathcal{K}}(\alpha + \beta \cdot t)} = \frac{1}{\alpha_i + \beta_i \cdot t} \tag{8}$$

The hazard rate in equation 8 correspond to the hazard rate of a Generalized Pareto Distribution (GPD), with scale parameter α_i and shape parameter β_i. In other words, when $v(t)$ is linear, the equilibrium distribution of selling times of each farmer follows a GPD.[12] Notice that this example also includes as a particular case the Exponential distri-

[12]The Generalized Pareto Distribution has a cumulative distribution function $F(t) = 1 + (1 + \beta_i t/\alpha)^{1/\beta_i}$

bution, *i.e.*, when $\beta = 0$ the value function is constant, the hazard rate is constant and the distribution of selling times is Exponential.

The GPD has the nice property that if a random variable t has a GPD, then the conditional distribution of $t - \tau$ given $t > \tau$ is also a GPD, with the same shape parameter β_i and a scale parameter equal to $\alpha_i^{'} = \alpha_i + \tau\beta_i$. This means that in every stage game, the distribution of selling times since the last sale, for each farmer follows a GPD. The intuition is simple. When one farmer sells the game is similar than the original game. In the original game we have a set of $\phi_{i\mathcal{K}}$ that is characteristic of each game and have one element for each farmer. In that game, looking at the future each farmer is playing a game where there is an immediate value of selling of $\alpha_i = \phi_{i\mathcal{K}}\alpha$ and a slope of $\beta_i = \phi_{i\mathcal{K}}\beta$, where $\mathcal{K}$ is the original set of farmers. If another farmer sells at time $t = \tau$, then each farmer is playing a game with an immediate value of selling of $\alpha_i^{'} = \phi_{i\mathcal{K}'}\left(\alpha + \beta\tau\right)$ and a slope of $\beta_i = \phi_{i\mathcal{K}'}\beta$, where $\mathcal{K}^{'}$ is the original set of farmers minus the farmer that sold. In other words, the time at which a stage game ends would affect the scale but not the shape of the distribution of sell times of subsequent stage games, and it would affect the scale parameter linearly. Given this structure, we only need to estimate two parameters α and β that would determine the shape and the original scale in the value function that the city offered to the farmers. We could estimate one pair for each ditch.

4.4 Identification

For simplicity, we drop the sub-index reflecting that in a given game the remaining player belongs to the set $\mathcal{K}$. From the previous section, we know that in a game of WoA the distribution of selling times are determined by the value of selling. Equation 1 defined the object of interest $\Delta_i(t)$ as the difference between the continuation value when another farmer sells $W_i^j\left(t\right)$ and the value of selling $V_i\left(t\right)$. In the empirical application, we only observe each farmer selling once, so we will not be able to estimate all ϕ_i. However, we can classify farmers depending on their observable characteristics, such that we will observe several selling times for a given configuration of the game. Therefore we can identify the function $v(t)$ non-parametrically. We will also observe all realizations of $V_i\left(t\right)$, which are the prices at which the farmers sold their plot. Therefore we are able to independently identify the functions $W_i\left(t\right)$ and $V_i\left(t\right)$. This means we could identify asymmetric values for each farmer, but not externalities. Finally, because we have information regarding the locations of the farmers' plots and their characteristics, we will be able to identify and estimate different functions $W_i\left(t\right)$, for different pairs of farmers i and j.

and a probability density function $f\left(t\right) = \alpha^{-1}\left(1 + \beta_i t/\alpha\right)^{1/\beta_i - 1}$.

In other words, if we only have information on selling times, as is usually the case (see Takahashi, 2015), then we could only identify $v(t)$, that is the probability of selling for a farmer in a particular game, and we would have to restrict attention to symmetric games, estimating a single ϕ for a given number of farmers, identified up to a constant. In this case the function $\Delta(t)$ is just equal to the hazard rate of the distribution of selling times for each game with the same number of farmers. That is, we could estimate a function for games with two farmers, another function for games with three farmers and so on.

If we also have information on the size of land and the value of the land for each farmer, then we could estimate an asymmetric WoA game and estimate $\Delta_i(t)$, thus identifying $v\,(t)$ and ϕ_i, up to a constant. If in addition, we have information on the prices received by the farmers, we could also estimate $W_i(t)$ from $V_i(t)$, thus identifying $v\,(t)$ and ϕ_i exactly. This is not trivial, and it is key in this case for both the estimation of the game and the counterfactuals. Moreover, it is rare to have such detailed data in an empirical estimation. Finally, if we have information regarding the locations of the farmers' plots and their characteristics, as well as the prices, we will be able to identify and estimate different functions $W_i^j(t)$, for different pairs of farmers, thus identifying $v\,(t)$ and ϕ_i exactly. Notice that this is the main innovation of the paper. We are estimating the externalities that a farmer exerts on another farmer when she sells her land. Depending on the variability of the data, and how we define a market (game) we could be more or less flexible on the structure of $W_i^j(t)$. Summarizing, we can identify

- **Symmetric Game** — Data on selling times: $\Delta(t)$.
- **Asymmetric Game** — Data on selling times and individual characteristics: $\Delta_i(t)$.
- **Asymmetric Game** — Data on selling times, individual characteristics and sale prices: $W_i(t)$ and $V_i(t)$.
- **Asymmetric Game with Externalities** — Data on selling times, individual characteristics, sale prices and pair-specific information: $W_i^j(t)$ and $V_i(t)$.

5 Estimation Strategy

In the data, some events would affect all farmers, not only farmers in the same ditch. The implicit assumption here is that we assume that sales by farmers outside the ditch affect all farmers in a given ditch in the same way. In particular, we will use the cumulative sales as a state variable in each game. In contrast, we believe that sales by farmers in the same ditch, will affect more farmers within the same ditch. Moreover, we think they could affect each farmer differently. Each stage game, as explained in Section 4, provides a selling time, which is the key variable. Each stage game also provides us with information regarding the farmer that sold, the farmers that were active but were not the first to sell, and the set of farmers that belonged to the same ditch, but have sold already.

The key econometric innovation concerning previous work relates to spatial externalities and time-varying values. As mentioned above, we have information regarding the location of each farmer's plot and the exact date of sale. This information is essential to estimate the spacial externalities and, therefore, test the checkerboarding claim. Moreover, preliminary evidence shows two relevant empirical facts: the hazard rate of selling times was not constant over time and sales of farmers on the same ditch affect the selling behavior of the remaining farmers. The former means that we need to model the dynamics of the farmer's behavior. The latter means that we need to model the externalities among farmers. Our econometric model can account for both facts.

We focus on a type of model, that features a unique equilibrium, and can account for the mentioned features of the data. This model falls within the class of proportional hazard rate models (PHRM). However, it is more general than the usual application of PHRM as we allow the baseline hazard, *i.e.*, the component of the hazard rate that depends on time only, to vary by game. In other words, we allow the baseline hazard rate to reset every time a farmer in the same ditch sells her property. Therefore, our model encompasses the Cox model in terms of the generality of the data generating process, being the Cox model a particular case of our model when the hazard rate does not reset. Moreover, in the main estimation, we assume the value functions to be linear over time. In this case, we show that the distribution of exit times follows a Generalized Pareto distribution.

The estimation consists of two steps. In the first step (Inner Loop), we get one vector θ^n of pseudo parameter for each game, with as many elements as farmers in that game. In the second step (Outer Loop), we use *hedonic* regressions to get a set of parameters β, using the pseudo-parameters we estimated in the first step. This allows us to separate both parts, and estimate the hedonic parameters without having to use simulations.

5.1 Proportional Hazard Rate Models

In Section 4 we used assumption A1 to solve the model. Assumption A1 implies that when looking at the empirical distribution of selling times for each farmer, their hazard rates $\lambda_{i\mathcal{K}}(t)$ are proportional to each other. Thus, we need to estimate a Proportional Hazard Rate Model (PHRM). The CDF of a PHRM is defined by

$$1 - F(t; \Omega; \theta) = [1 - G(t; \Omega)]^{\theta}, \theta > 0, \tag{9}$$

and we write the PDF as

$$f(t; \Omega; \theta) = \theta g(t; \Omega) [1 - G(t; \Omega)]^{\theta - 1}, \theta > 0, \tag{10}$$

where $G(t; \Omega)$ is a CDF, θ is a positive shape parameter and Ω is a parameter vector that characterizes the source distribution. The hazard rate of a PHRM distribution is $\frac{f}{1-F} = \frac{\theta g[1-G]^{\theta-1}}{[1-G]^{\theta}} = \theta \frac{g}{1-G}$ which implies that the hazard rate of F is proportional to that of G, and the scalar of proportionality is the shape parameter θ. In both cases, we call $G(x; \Omega)$ the source CDF, due to the generation process. This class of models is interesting because, as shown above, a WoA with changing values will generate this statistical process. The equilibrium strategies for each farmer will be to choose a selling time. The probability distribution of selling times for a given farmer in a given stage game will then follow a PHRM distribution. All farmers will have the same source distribution, which is determined by $v(t)$ but a different shape parameter $1/\phi_{i\mathcal{K}}$.

There is, however, one issue when linking the model to the data. Even when the equilibrium strategy for each farmer consists on choosing a selling time for each stage game, we would only observe the selling time for the farmer who sells, *i.e.*, the farmer whose time of sale is the lowest. In other words, we have a censored problem. For the other farmers we can only infer that their selling times were larger. This is similar to the issue of estimating the distribution of valuations in a second price auction (SPA) when the econometrician only observes the winning bid. In a SPA, the observed behavior (winning bid) is the second order statistic of the underlying distribution of valuations. In the WoA, the observed behavior (selling time) is the minimum of the underlying distribution of selling times. Moreover, in our case, it is the minimum of asymmetric random variables, because the farmers have different valuations for staying or selling in a given stage game. The properties of the PHRM are useful when dealing with this issue.

Following Espín-Sánchez and Wu (2019) we have also developed the distribution of order statistics for an asymmetric sample. Let $T_1, ..., T_n$ be a random sample of size n where each realization comes from a PHRM with different parameters $\Theta \equiv (\theta_1, ..., \theta_n)$. In particular, $T_i \sim G(\theta_i)$ for $i = 1, 2, ..., n$, that is $T_1 \sim PHRM(\theta_1)$, $T_2 \sim PHRM(\theta_n)$,..., $T_n \sim PHRM(\theta_n)$. In this case, we can compute the order statistics of the asymmetric random sample. In particular, here we are interested in the first asymmetric order statistic (minimum) which has the form

$$f_1(t) = f\left(t; \Omega; \bar{\theta}\right), \tag{11}$$

where $f(t; \Omega; \theta)$ is the density function of a PHRM and $\bar{\theta} = \sum_{k=1}^{n} \theta_k$. Notice that in the symmetric case there $\theta_i = \theta$, equation 11 becomes simpler. In particular, the distribution of the minimum for the symmetric case is just that of a PHRM with parameter $n\theta$.

In our baseline case, as shown above in subsection 4.3, we assume that the value function is linear over time and, thus, the distribution of selling times follows a Generalized Pareto Distribution (GPD), where $\Omega \equiv (\alpha, \beta)$. A PHRM with GPD is characterized by

$$f(t; \alpha, \beta; \theta) = \frac{\theta}{\alpha} (1 + \beta t/\alpha)^{(\theta - \beta)/\beta} \tag{12}$$

and a cumulative distribution function

$$1 - F(t; \alpha, \beta; \theta) = (1 + \beta t/\alpha)^{\theta/\beta} \tag{13}$$

The hazard function is then

$$h(t; \alpha, \beta; \theta) = \frac{f(t; \alpha, \beta; \theta)}{1 - F(t; \alpha, \beta; \theta)} = \frac{\frac{\theta}{\alpha} (1 + \beta t/\alpha)^{(\theta - \beta)/\beta}}{(1 + \beta t/\alpha)^{\theta/\beta}} = \theta \frac{1}{\alpha + \beta \cdot t} \tag{14}$$

We can then characterize equation 11 for the case where the source distribution follows a GPD as

$$f_1(t; \alpha, \beta; \theta) = \frac{\bar{\theta}}{\alpha} (1 + \beta t/\alpha)^{(\bar{\theta} - \beta)/\beta} \tag{15}$$

A PHRM is characterized empirically by a separability assumption, *i.e.*, we can decompose the hazard rate into two independent components: a baseline hazard rate that depends on time but is common across individuals and an idiosyncratic component that does not change over time. Therefore, we can write the empirical hazard function as

$$h(t, x) = h_0(t) \cdot \theta(X_i, \beta) \tag{16}$$

where $h_0(t)$ is the baseline hazard rate and $\theta(X_i, \beta)$ is the idiosyncratic component. The vector X_i should not change over time. However, in our case each observation is a particular stage-game that begins when one farmer sells and ends when the next farmer sells. Therefore we can include variables in X_i that change over time, as long as they do not change during the stage game, *i.e.*, we can include state variables that change over time such as the percentage of farmers in a ditch that has already sold, or the fraction of water rights remaining in a given ditch.

5.2 First Step

In the first step of the estimation, we recover a θ for each farmer. We allow farmers to be heterogeneous concerning their "shape" parameter. To do so, we consider that each ditch is an independent game and thus estimate a vector of shape parameters for each game independently. We have thirteen ditches that feature at least six sales from farmers in that ditch. For each ditch, we order farmers as a function of their selling time, and we calculate the number of days between farmers' sales.

Using only selling times, we calculate the probability that a given farmers sell in x days when there are n remaining farmers in a game. In such a game, the strategy for each farmer is to sell at each point in time using an instantaneous probability $\eta^n(t)$. However, we do not observe all the realizations of sell times. We only observe the lowest among all realizations, that is the minimum or the first order statistic.[13] Therefore, the distribution of sell times would follow the distribution of the sell times of the first order statistic. We can also define $\Psi^n(x;\theta^n)$, with density $\psi^n(x;\theta^n)$, as the distribution of the first order statistic (minimum) or n draws from of $\Psi^n(t;\theta^n)$. In other words, each farmer will draw a time of selling t_i from an $EG(\theta^n)$ but we will only observe the sell of the farmer with the lowest realization.

To estimate an asymmetric game with n farmers, we use the following likelihood function

$$l\left(T_i^n, \theta^n\right) = \prod_{i=1} \psi^n(x_i^n;\theta^n) = \prod_{i=1} \left\{f_{1:n}(x_i^n;\theta^n)\right\} \tag{17}$$

where x_i is the realization of number of days until the sale, since the beginning of the game, in a game with n remaining farmers, and $f_{1:n}(x_i^n;\theta^n)$ is the density of the minimum as defined in equation (8). We estimate our vector of parameters running a Maximum Likelihood Estimator (MLE) for each game.

giving us an estimate for θ^n that allows us to calculate the probability that any farmer sells and also the continuation value for each farmer $\Delta^n(t^n)$.

Given the functional forms derived from the theory, we can build continuation values using our estimated hazard rates.

In particular, the theoretical results imply that $h\left(x^n, \theta^n\right) = \eta^n(x^n) = \frac{1}{(n-1)\Delta^n(x^n)}$, for all

[13] Remember that a War of Attrition can be modeled as a particular all pay auction, where all n farmers pay the lowest bid, and the $n-1$ farmers with the highest bids get the prize, that is, they get to stay in the game. In that analogy, the waiting time is the War of Attrition game is equivalent to the bid in the all pay auction.

games. Therefore, since $\Delta^n(x^n) = \frac{1}{(n-1)h(x^n,\theta^n)}$ with the estimated value $\hat{\theta}^n$ we can compute the estimated hazard function $h\left(x, \hat{\theta}^n\right)$. and thus recover the distribution of valuations for each game $\Delta^n(x)$, which is equal to

$$\Delta^n(x) = \frac{1}{(n-1)h\left(x, \hat{\theta}^n\right)}$$

The advantage of using a PHRM is that we can compute what would be the probability of the minimum for each game making the likelihood estimation computationally feasible.

5.3 Second Step

From the previous section we know that in a game of WoA the distribution of selling times are determined by the value of selling. Moreover, in equilibrium, the hazard function of the selling distribution for a particular game with a set of n farmers is identical to the difference in continuation values $\Delta_i(t)$, where

$$\Delta_i(t) \equiv \sum_{j \neq i} p_j(t) W_i^j(t) - V_i(t) = W_i(t) - V_i(t) \tag{18}$$

where $p_j(t)$ is the instantaneous probability that farmer j sells at time t, $W_i^j(t)$ is the continuation value of a game where farmer j has just sold and $V_i(t)$ is the value that farmer i gets by selling. Since valuations are restricted to be positive, we define $V_i(t) = ln(P_i(t))$. This way prices are the exponential of the intrinsic valuation.Notice that while $W_i^j(t)$ and $V_i(t)$ are fundamentals of the model, p_j is an equilibrium outcome that will be determined with similar equations for the other farmers, that is using $\Delta_j(t)$. Notice that as we observe several selling times for a given configuration of the game, we have already identified the function $\Delta_i(t)$ non-parametrically. We will also observe several realizations of $V_i(t)$ which are the natural log of the price at which the farmers sold their plot. Therefore can independently identify the functions $W_i(t)$ and $V_i(t)$. Finally, since we have information regarding the locations of the farmers' plots and their characteristics, we will be able to identify and estimate different functions $W_i^j(t)$, for different pairs of farmers. We can thus re-write the equation as

$$V_i(t) = \sum_{j \neq i} p_j(t) W_i^j(t) - \Delta_i(t) \tag{19}$$

Here we can observe prices, and we already have an estimate for the probability of any farmer selling, and their continuation values. We will need to make one further assumption to be able to identify the potential externalities.

Given that we need to estimate a vast number of parameters, and the fact that we only observe certain sales as each game is played only once, then we will need to assume a parametric function for the counterfactual estimation values. We will assume that we can decompose such value as the linear combination of relative observable characteristics between i and j.

$$W_j^i = \beta_1 X_{ij}^1(x_i) + \beta_2 X_{ij}^2(x_i) + .. + \beta_K X_{ij}^K(x_i) = \beta_{1\times K} X_{K\times 1}^{ij}$$

Where we have K hedonic characteristics we will use, and $< X_{ij}^1 = g(X_i^1, X_j^1, x_i)..$ $X_{ij}^K = g(X_i^K, X_j^K, x_i) >$ is some function of time-varying attributes J (or the attribute in time x_i) for i and j (this could be the difference, yet we could also use a more flexible specification).We are interested in recovering $\beta_{1\times K} \equiv < \beta_1 ... \beta_K >$. In Appendix A.1 we show we can write the above system as system as

$$\beta_{1\times K}\cdot \begin{array}{c} \left[\hat{p}_1^2(x_2:\hat{\theta})X_{2j}^1(x_2), .., \hat{p}_1^2(x_2:\hat{\theta})X_{2j}^K(x_2)\right]_{K\times 1} \\ [...] \\ \beta_{1\times K}\cdot\left[\sum_{j<N}\hat{p}_j^N(x_N:\hat{\theta})X_{Nj}^1(x_N), .., \sum_{j<N}\hat{p}_j^N(x_N:\hat{\theta})X_{Nj}^K(x_N)\right]_{K\times 1} \end{array} - \Delta_{Nx1} = V_{N\times 1}$$

We define

$$\hat{M}_{K\times N} = \begin{array}{c} \left[\hat{p}_1^2(x_2:\hat{\theta})X_{2j}^1(x_2), .., \hat{p}_1^2(x_2:\hat{\theta})X_{2j}^K(x_2)\right]_{K\times 1} \\ [...] \\ \beta_{1\times K}\cdot\left[\sum_{j<N}\hat{p}_j^N(x_N:\hat{\theta})X_{Nj}^1(x_N), .., \sum_{j<N}\hat{p}_j^N(x_N:\hat{\theta})X_{Nj}^K(x_N)\right] K\times 1 \end{array}$$

$M_{K\times N}$ is a matrix that has as many columns as hedonic characteristics, and as many rows as farmers in a ditch. This matrix represents the weighted average (weighting by probability) of relative hedonic characteristics. What is crucial, is that we can compute this matrix, since we can compute the probabilities, and we observe relative characteristics. Then we have the following linear system

$$V_{N\times 1} = \beta_{1\times K}\hat{M}_{K\times N} - \Delta_{Nx1}$$

Hence from this expression, we get that we can recover the fundamental structural parameters of the game $\hat{\beta}$, running a simple linear regression of the form

$$V_{N\times 1} = \beta_{1\times K}\hat{M}_{K\times N} - \Delta_{Nx1} + \varepsilon$$

Notice that the above expression is very flexible. We can estimate the same regression with a random coefficients model, allowing for the hedonic parameters to be ditch dependent.

Table 7 reports the estimated value, for the linear specification that allows us to recover the structural parameters of the game. It is crucial to note that in the hedonic model, there are still many degrees of freedom regarding the econometric specification of the linear regression. To have a criterion for variables to include and to exclude, we use a machine learning method (Random Forest, RF). In the RF we try to predict prices as a function of all possible observable variables, including plot characteristics as well as the externality variables defined above. We then evaluate variables in terms of their explanatory power and include in the final hedonic specification only the variables that had high explanatory power.

In Table 7, we see the results for the regression running both an OLS regression as well as random coefficients models, where we allow betas to vary by a ditch. "Days" represents days from first sale. It is suppose to capture any time varying trend. "Min distance" represents the distance to the river. "Value asymmetric" is the idiosyncratic component of the continuation value for a farmer that does not sell calculated from the model, assuming an asymmetric probability distribution. "Crop_p" represent the cultivated area for each farmer, as a percentage of the total cultivated area in a given ditch (this allows us to pool ditches, and make farmers comparable across games). "Area acres_p" and "water acres _p" also represent percentage of total acres, and percentage of total water acres per ditch. We find that distance to the river plays a crucial role in the price farmers received. The closer to the river, the higher the price you can charge for your land. On the other hand, the more land, water acres farmers had, the more they got for their property.

All the variables termed "others," are the weighted average (using the MLE probabilities), of the characteristics of the rest of the farmers that remain on a ditch for each player. These variables aim to capture the externalities that other farmers had on the selling prices, by game. We see that the closer a farmer it is to you, the higher the externalities she imposes on your selling price when she sells her property. Farmers with more land, also tend to impose bigger price externalities on not selling farmers.

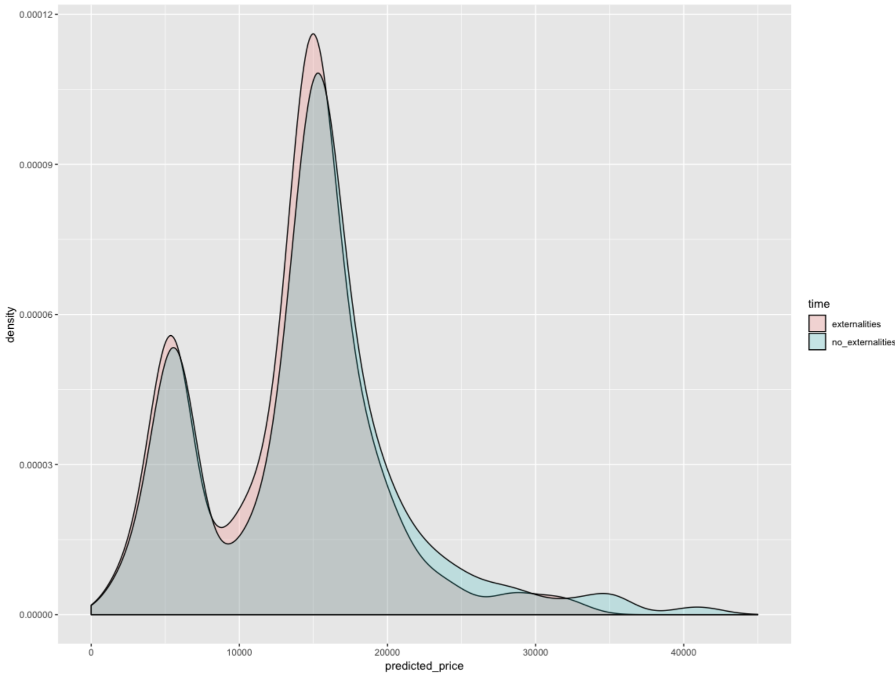
6 Counterfactuals

Once we estimate the structural model above, we can compute the price for all farmers, for any potential specification of the model, as the following expression gives expected prices

Table 7: Structural Estimates.

	Dependent variable:	
	price	
	OLS	*linear mixed-effects*
	(1)	(2)
value_asymetric	6,841.813	−20,392.340
	(44,337.630)	(25,612.750)
days	−22.269***	1.009
	(6.242)	(4.146)
min_river	−7.935**	−7.395**
	(3.543)	(3.044)
areaacres_p	637,029.000***	551,742.400***
	(182,223.700)	(121,374.900)
water_acre_p	209,889.000**	2,416.525
	(87,075.660)	(57,286.500)
crops_p	−447,093.200***	−86,807.920
	(166,911.600)	(116,811.900)
distance_other	−0.589**	−0.765
	(0.293)	(8.883)
days_since_other	27.521***	5.147
	(6.895)	(888.192)
acres_other	−428,665.800***	−67,330.990
	(105,160.400)	(75,834.610)
Constant	246,543.300***	7,777.345
	(59,837.140)	(39,745.890)
Observations	328	328
R^2	0.385	
Adjusted R^2	0.367	
Log Likelihood		−3,851.471
Akaike Inf. Crit.		7,766.942
Bayesian Inf. Crit.		7,888.318
Residual Std. Error	63,071.120 (df = 318)	
F Statistic	22.075*** (df = 9; 318)	
Note:		*p<0.1; **p<0.05; ***p<0.01

Figure 5: Counterfactual distribution of prices, with and without externalities.



Notes: This figure shows the expected distribution of prices the model predicts with and without externalities.

$$P_{N\times 1} = e^{\hat{\beta}_{1\times K}\hat{M}_{K\times N} - \hat{\Delta}_{Nx1}} \tag{20}$$

Notice that in equation 20, regardless of the estimated parameters, prices are bounded below by zero. Therefore, we can estimate the distribution of prices the city would have paid, in the absence of externalities. To do so, we run a counterfactual exercise were we compute expected prices in the presence of externalities, vis-à-vis expected prices when the coefficients associated to the sale of other farmers in the ditch are equal to zero. We find that on average, the city pays 7.6 percent less because of externalities. On the other hand, the presence of externalities compressed prices, featuring a standard deviation of 7.2 percent higher than in the case when we shut down those forces.

We then compute the difference between the price a farmer received would received with externalities and the price a farmer would received without externalities. We termed this difference "gains from externalities," and we plot the difference for each farmer against farmers characteristics in Figure 6 below. In Figure 6.A we show gains from externalities in the y-axis versus the amount of cultivated land each farmer had in the x-axis. We see that there is a non-monotonic relation between gains form externalities and the acres of cultivated land. Farmers that had more cultivated land were more exposed to financial loses given externalities in the bargaining process. Figure 6.B shows a similar scatter plot with gains from externalities and total acres. We see a very similar non-monotonic relationship between the two variables. This suggests that whether a farmer have more of its land cultivated or not played little role on the externalities received. Results are also similar regarding the number of water acres farmers had, as shown in Figure 6.C. These three relations look similar should come at no surprise, as the three variables are positively correlated. Finally, Figure 6.D shows a scatter plot showing gains from externalities versus distance to the river. The farmers that gain the most, in terms for the price received form the city, from externalities were the ones that were closer to the river. We can see a significant negative relationship up to a about one kilometer, where this relationship flattens out.

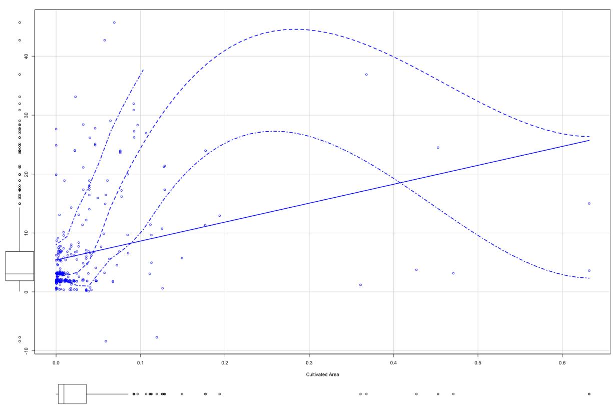
7 Conclusions

In this article, we have explored the most famous and controversial water transfer in the history of the U.S., that water transfer from the Owens Valley to Los Angeles. The main accusation against the city was that the city was "checkerboarding," *i.e.*, that it targeted particular farmers to make other farmers more willing to sell or to sell at a lower price. In economic terms, the accusation implies that there were spatial externalities among neighboring farmers or farmers in the same ditch. This raises several questions. Were there spatial externalities in the Owens Valley? If so, how important were those externalities for the prices that the city finally paid? And, did the city, knowing of these externalities, acted strategically to target key farmers?

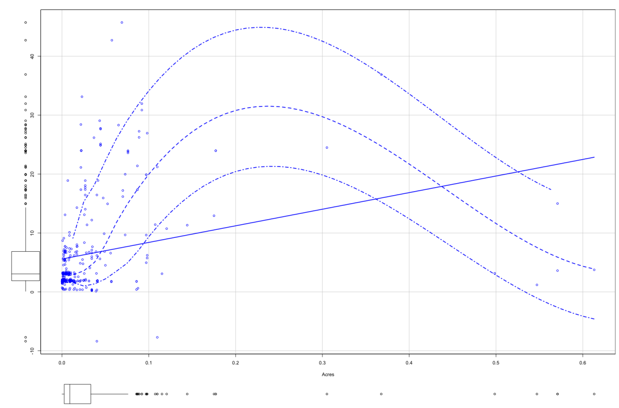
We collected new data regarding the price and date of each sale, as well as their geographical location and other characteristics, to address the questions above. The key innovation concerning previous work is to have information regarding the location of each farmer's plot and exact date of sale. This is essential to estimate the spatial externalities and, therefore, test the checkerboarding accusation. Moreover, preliminary evidence shows two relevant empirical facts: hazard rate of selling times was not constant over time and sales of farmers on the same ditch affect the selling behavior of the remaining farmers. The former means that we need to model the dynamics of the farmer's behavior. The latter means that we need to model the externalities among farmers. We present a new econometric model that can account for both facts, and that have a unique equilibrium consistent with the data. The model falls within the class of proportional hazard rate models (PHRM), but it is more general than the Cox model in terms of the generality of the data generating process. In particular, whereas Cox requires the baseline hazard rate

Figure 6: Gains from externalities.

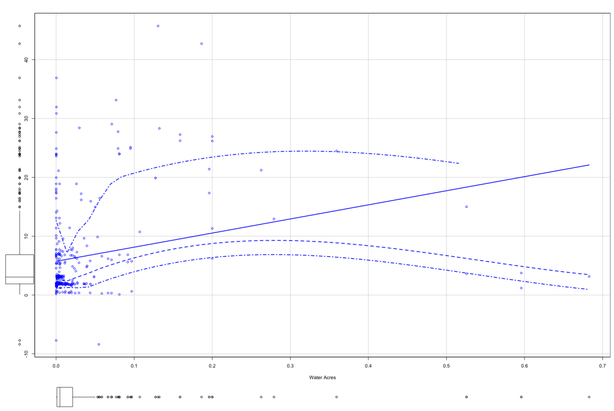
A. Cultivated Land.



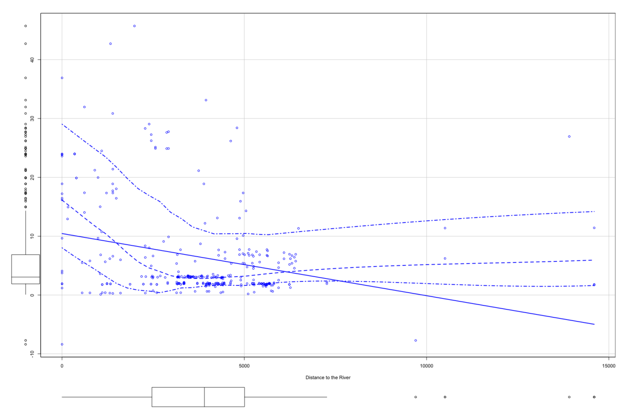
B. Acres.



C. Water Acres.



D. Distance to the River.



Notes: The solid line follows a linear fit, whereas the doted line specifies a non-parametric estimate of the mean or median function of the vertical axis variable given the horizontal axis variable and optionally a non-parametric estimate of the conditional variance. Both the mean function and variance functions are drawn for ungrouped data

Panel A: scatter plot of gains from externalities, and cultivated land. Panel B: scatter plot of gains from externalities, and acres. Panel C: scatter plot of gains from externalities, and water acres. Panel D: scatter plot of gains from externalities, and distance to the river.

to the common to all farmers, our model allows for the hazard rate to "reset" every time a farmer exits. The Cox model is then a particular case of our model when the hazard rate does not reset. Moreover, when we assume that the value functions are linear concerning time, we show that the distribution of exit times follows a Generalized Pareto distribution.

In addition to the new econometric model, we present an original estimation method for such a model. The estimator proceeds in two stages and makes the estimation simple and faster and making the estimator transparent and very flexible in terms of variables and specifications. In the first stage, we estimate the pseudo-parameters that are consistent with the behavior observed in the data. These pseudo-parameters are consistent with any specification regarding the relationship between the individual characteristics of the farmers, and the externalities, and are only a function of the exit times. Therefore, they only need to be estimated once. These pseudo-parameters are also easy to estimate due to a key result. The result derives from the fact that in PHRM, the asymmetric order statistic of the minimum has a closed-form solution. In the second stage, we can use the estimated pseudo-parameters and estimate the relationship between the farmers' and neighbors' characteristics and prices. The specification in the second stage is unrestricted and makes the results easy to interpret and transparent. The intuition behind these results is more general than the case studied here and could be applied to other settings. The results are easily applicable to environments with information externalities or where PHRM are suitable.

Our results show that there were critical spatial externalities, but the city did not generally base their offers on the potential externalities that the farmers would pay, but instead on the fundamental value of the land. The city, however, targeted particular farmers in 1922, which did create significant externalities, and effectively broke the coordination problem, by disrupting the farmers selling association. In this sense, the city did not checkerboard farmers systematically because after having bought the key farmers from the main ditches, the farmer's coordinating threat was effectively over. This insight is a result of a combination of detailed historical research—by identifying the claims made by historians in the past and looking closely at the data—and rigorous analytical economic analysis—by collecting comprehensive data and developing new models and estimation techniques to address the problem. We think that this combination would be useful in many other historical settings.

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A Structural Estimation

A.1 Derivation of Linear Regression for the Structural Model

We can write the system of equations that we need as

$$\begin{gathered}\sum_{j<2}\hat{p}_j^2(x_2:\hat{\theta})W_j^2(x_2)-\Delta_i^2(t)=V^2(x_2)\\ \sum_{j<3}\hat{p}_j^3(x_3:\hat{\theta})W_j^3(x_3)-\Delta_i^3(t)=V^3(x_3)\\ ....\\ \sum_{j<N}\hat{p}_j^N(x_N:\hat{\theta})W_j^N(x_N)-\Delta_i^N(t)=V^N(x_N)\end{gathered}$$

Where N is the maximum number of farmers in a given ditch.

Note that this system can be re-written as

$$\begin{gathered}\sum_{j<2}\hat{p}_j^2(x_2:\hat{\theta})\beta_{1\times K}X_{K\times 1}^{2j}-\Delta_i^2(t)=V^2(x_2)\\ \sum_{j<3}\hat{p}_j^3(x_3:\hat{\theta})\beta_{1\times K}X_{K\times 1}^{3j}-\Delta_i^3(t)=V^3(x_3)\\ ....\\ \sum_{j<N}\hat{p}_j^n(x_N:\hat{\theta})\beta_{1\times K}X_{K\times 1}^{nj}-\Delta_i^N(t)=V^N(x_N)\end{gathered}$$

Notice that we can rearrange terms here. Rearranging we get

$$\sum_{j<2}\hat{p}_j^2(x_2:\hat{\theta})\beta_{1\times K}X_{K\times 1}^{2j}=\hat{p}_1^2(x_2:\hat{\theta})\left[\beta_1X_{2j}^1(x_2)+\beta_2X_{2j}^2(x_2)+..+\beta_KX_{2j}^K(x_2)\right]$$

$$=\beta_{1\times K}\cdot\left[\hat{p}_1^2(x_2:\hat{\theta})X_{2j}^1(x_2),..,\hat{p}_1^2(x_2:\hat{\theta})X_{2j}^K(x_2)\right]_{K\times 1}$$

$$\sum_{j<3}\hat{p}_j^3(x_3:\hat{\theta})\beta_{1\times K}X_{K\times 1}^{3j}=$$

$$\hat{p}_1^3(x_3:\hat{\theta})[\beta_1X_{3j}^1(x_3)+\beta_2X_{3j}^2(x_3)+..+\beta_KX_{3j}^K(x_3)]+\hat{p}_2^3(x_3:\hat{\theta})\left[\beta_1X_{3j}^1(x_3)+\beta_2X_{3j}^2(x_3)+..+\beta_KX_{3j}^K(x_3)\right]$$

$$=\beta_{1\times K}\cdot\left[\sum_{j<3}\hat{p}_j^3X_{3j}^1(x_3),..,\sum_{j<3}\hat{p}_j^3X_{3j}^K(x_3)\right]_{K\times 1}$$

In general we will have that

$$\sum_{j<N} \hat{p}_j^N(x_N : \hat{\theta})\beta_{1\times K} X_{K\times 1}^{Nj} = \beta_{1\times K} \cdot \left[\sum_{j<N} \hat{p}_j^N X_{Nj}^1(x_N), .., \sum_{j<N} \hat{p}_j^N X_{Nj}^K(x_N)\right]_{K\times 1}$$

Therefore we can rewrite the system as

$$\beta_{1\times K} \cdot \begin{matrix} \left[\hat{p}_1^2(x_2 : \hat{\theta}) X_{2j}^1(x_2), .., \hat{p}_1^2(x_2 : \hat{\theta}) X_{2j}^K(x_2)\right]_{K\times 1} \\ [...] \\ \beta_{1\times K} \cdot \left[\sum_{j<N} \hat{p}_j^N(x_N : \hat{\theta}) X_{Nj}^1(x_N), .., \sum_{j<N} \hat{p}_j^N(x_N : \hat{\theta}) X_{Nj}^K(x_N)\right]_{K\times 1} \end{matrix} - \Delta_{Nx1} = V_{N\times 1}$$

B War of Attrition with 3 players

In this section, we fully solve the War of Attrition (WoA) game with three players, with time-unvarying values, for the symmetric and asymmetric case, as well as the more general case with externalities. The goal of this section is to get the intuition of the equilibrium. We also develop some results that we will use in Section C to discuss identification results.

B.1 Symmetric Game

Just three parameters can characterize a symmetric game: $V^3 < V^2 < V^1$. Where V^l is the value that a player gets from getting the l^{th} prize. The game is solved by backward induction, but since all players are identical, any of the possible sets of two players in the second stage will be identical. Moreover, both players in that game would have the same valuations, so the game is the standard two-player symmetric WoA. In that case the equilibrium is found by solving the indifferent condition for player i

$$\lambda_j^2 \left(V^1 - V^2\right) dt = 1dt$$

where λ_j^l is the instantaneous probability (or hazard rate) that player j will exit, when he would get the l^{th} prize. The left-hand side of the equation represents the instantaneous benefits of staying, while the right-hand side represents the instantaneous costs of staying. Because the values do not change over time, until a player exits, the hazard rate of exiting cannot change over time either. It is worth noticing that the valuations of his opponents completely determine the strategy of player j. In this case, due to symmetry, the hazard function of each player is the same. Therefore $\lambda_i = \lambda_j = 1/(V^1 - V^2)$. It is also worth noticing that this equilibrium exhausts all the rents for all players. This means that the expected utility of playing this game for each player is just V^2 that is, the utility they would get if they exit right away. Each player is imposing a negative externality on each other by staying in the game. The externalities are maximal in the sense that they fully dissipate all the rents for the other player.

We can now focus on the game with three players. In this case the indifferent condition for player i is

$$\lambda_j^3 \left(W^2 - V^3\right) dt + \lambda_k^3 \left(W^2 - V^3\right) dt = 1dt$$

where W^2 is the continuation value for player i if another player exits. The first term in the left-hand side represents the hazard rate that player j will exit times the benefit to player i if that happens, the second term in the left-hand side represents the hazard rate that player k will exit times the benefit to player i if that happens and the right-hand side is the cost. Notice that now solving the game is a little harder, because we have three equations with three unknowns but, unlike in the case with two players, there is more than one unknown in each equation. However, due to symmetry, it is easy to see that $\lambda_i^3 = \lambda_j^3 = \lambda_k^3$ and then all three equations are identical, and we can use one of them to solve the game and see that $\lambda_i^3 = \lambda_j^3 = \lambda_k^3 = 1/2(W^2 - V^3)$. As we have seen before, the continuation game in the symmetric case is the same regardless of the identity of the player who exits. Moreover, the continuation value is the value of receiving the second prize, *i.e.*, $W^2 = V^2$. Therefore, $\lambda_i^3 = \lambda_j^3 = \lambda_k^3 = 1/2(V^2 - V^3)$. This is remarkable because it means that the strategies for all players in the first game are independent of the valuations the players assigned to the last prize V^1.

B.2 Asymmetric Game

An asymmetric game can be characterized by nine parameters: $V_i^3 < V_i^2 < V_i^1$, for $i = 1, 2, 3$. Where V_i^l is the value that player i gets from getting the l^{th} prize. The game is solved by backward induction. Since not all players are identical any of the possible sets of two players in the second stage will not be identical. Both players in that game would not have the same valuations, but the game is the standard two-player asymmetric WoA. In that case the equilibrium is found by solving the indifferent condition for players i and j is

$$\begin{aligned}
\lambda_j^{2,i}\left(V_i^1 - V_i^2\right) dt &= 1dt \\
\lambda_i^{2,j}\left(V_j^1 - V_j^2\right) dt &= 1dt
\end{aligned}$$

where $\lambda_j^{2,i}$ is the hazard rate that player j will exit, when she is playing against player i and she would get the second prize. The left-hand side of the equation represents the benefits of staying, while the right-hand side represents the costs of staying. Because the values do not change over time, until a player exits, the hazard rate of exiting cannot change over time either. It is worth noticing that the valuations of his opponents completely determine the strategy of player j. In this case, due to asymmetry, the hazard rate of each player is not the same, but it can be easily computed: $\lambda_i^{2,j} = 1/(V_j^1 - V_j^2)$ and $\lambda_j^{2,i} = 1/(V_i^1 - V_i^2)$. Notice that, even though we are computing six hazard rates (two for each of the three two-player games), some of them are going to be identical pairwise. In particular, $\lambda_i^{2,j} = \lambda_k^{2,j} = 1/(V_j^1 - V_j^2)$. That is, when either player i or player k are facing player j in the last round, they will exit with the same hazard rate. That means we only have to compute three hazard rates.

It is also worth noticing that this equilibrium exhausts all the rents for all players. This means that the expected utility for player i of playing this game is just V_i^2. That is, the utility they would get if they just exit right away. Notice that this value, is independent of the identity of the opponent, while the strategy and therefore the distribution of exit time is not. Each player is imposing a negative externality on each other by staying in the game. The externalities are maximal in the sense that they fully dissipate all the rents for the other player. In the asymmetric case, the larger the difference in valuations of my opponent, the lower is my exit rate, therefore the longer I stay and the larger is the externality I impose on my opponent. That is, in order to keep the value of playing the game constant, the larger is the differences in values of my opponent, the longer I need to stay to dissipate rents.

We can now focus on the game with three players. In this case the indifferent conditions for player i, j and k respectively are

$$\begin{aligned}
\lambda_j^3\left(W_i^{2,j} - V_i^3\right) dt + \lambda_k^3\left(W_i^{2,k} - V_i^3\right) dt &= 1dt \\
\lambda_i^3\left(W_j^{2,i} - V_j^3\right) dt + \lambda_k^3\left(W_j^{2,k} - V_j^3\right) dt &= 1dt \\
\lambda_i^3\left(W_k^{2,i} - V_k^3\right) dt + \lambda_j^3\left(W_k^{2,j} - V_k^3\right) dt &= 1dt
\end{aligned} \tag{21}$$

where $W_i^{2,j}$ is the continuation value for player i if player j exits. In the first equation, the first term in the left-hand side represents the hazard rate that player j will exit times the benefit to player i if that happens, the second term in the left-hand side represents the hazard rate that player k will exit times the benefit to player i if that happens and the right-hand side is the cost. The second and third equations are analogous. Notice that now solving the game is a little harder, because we have three equations with three unknowns but, unlike in the case with two players, there is more than one unknown in each equation. Moreover, due to asymmetry, in general the exit times will be different for each player. The system of equations now seems complicated. We need to solve for three variables $\lambda_i^3, \lambda_j^3, \lambda_k^3$ as a function of nine parameters, three in each equation. As we have seen before, the continuation game in the asymmetric case is the same regardless of the identity of the player who exits. Moreover, the continuation value is the value of receiving the second prize, *i.e.*, $W_i^{2,j} = W_i^{2,k} = V_i^2$. Therefore, there are only six parameters to take into account. We will not go into the details, but the solution to the system is

$$\begin{aligned}
\lambda_i^3 &= \tfrac{1}{2}\left(\tfrac{1}{V_j^2-V_j^3} + \tfrac{1}{V_k^2-V_k^3} - \tfrac{1}{V_i^2-V_i^3}\right)\\
\lambda_j^3 &= \tfrac{1}{2}\left(\tfrac{1}{V_i^2-V_i^3} + \tfrac{1}{V_k^2-V_k^3} - \tfrac{1}{V_j^2-V_j^3}\right)\\
\lambda_k^3 &= \tfrac{1}{2}\left(\tfrac{1}{V_j^2-V_j^3} + \tfrac{1}{V_i^2-V_i^3} - \tfrac{1}{V_k^2-V_k^3}\right)
\end{aligned} \tag{22}$$

although the formula seems complicated, it is easy to explain. The hazard rate of exit is always an "average" of the inverse of the differences in valuations for all the players in the game. The "weights" would be different for each player. In particular, the hazard rate for player i has positive weights on the elements from other players and a negative weight on her own element.[14]

We can simplify the notation, by defining $\Delta_i \equiv V_i^2 - V_i^3$, with this normalization there are only three parameters to take into account on this game. Each equation can be written as

$$\lambda_i^3 = \frac{1}{2}\left(\frac{1}{\Delta_j} + \frac{1}{\Delta_k} - \frac{1}{\Delta_i}\right) = \frac{1}{2}\left(-\frac{1}{\Delta_i} + \sum_{j\neq i}\frac{1}{\Delta_j}\right) \tag{23}$$

We have solved the system 21 above by solving for the values of λ as a function of Δ. This is what we need to do, from a theorist point of view, when we try to find the equilibrium strategies taken by each player, as a function of the parameters (values) that they observe. From the point of view of the econometrician, we still need to solve the system 21 above. However, from the econometrician point of view, we observe the hazard rate of exit (moreover, we observe the whole distribution of exit times), but we want to know the values of exiting for each player. Notice that this is a much simpler task since Δ_i only appears in the first equation, Δ_j only appears in the second equation and so on.

[14]It can be shown that this equilibrium, where all players mix all the time, exist if and only if all the probabilities are positive. This happen whenever the difference in valuations for any given player are not very different from the differences in valuations of all the other players.

Therefore, the solution for each parameter Δ_i is

$$V_i^2 - V_i^3 \equiv \Delta_i = \frac{1}{\lambda_j^3 + \lambda_k^3}$$

B.3 Asymmetric Game with externalities

An asymmetric game with externalities can be characterized by eighteen parameters: $V_i^{123}, V_i^{132}, V_i^{213}, V_i^{23}$ for $i = 1, 2, 3$. Where V_i^{xyz} is the value that player i gets when the allocation of prizes is such that player x gets the first prize, player y gets the second prize and player z gets the third prize. In the case we are considering here, however, we can restrict the number of parameters by having some restrictions on the vector of preferences for each player.

The first restriction is that after a player has exited, his valuation is not affected by the identities of the remaining players. In practice that means that $V_i^{iyz} = V_i^{izy}$, in our case with three players this means that the equilibrium could be characterized with fifteen parameters: $V_1^{1\cdot\cdot}, V_1^{213}, V_1^{231}, V_1^{312}, V_1^{321}$, $V_2^{123}, V_2^{132}, V_2^{2\cdot\cdot}, V_2^{312}, V_2^{321}$ and $V_3^{123}, V_3^{132}, V_3^{213}, V_3^{231}, V_3^{3\cdot\cdot}$.

The game is solved by backward induction. Since not all players are identical any of the possible sets of two players in the second stage will not be identical. Both players in that game would not have the same valuations, but the game is the standard two-player asymmetric WoA. Notice, however, that because we are in a three-player game, even if we have to solve each of the three two-player games independently, in each of them, there is only one possibility for the player who has already left. In that case, when player k has exited, the equilibrium is found by solving the indifferent condition for players i and j is

$$\begin{aligned} \lambda_j^{2,i}\left(V_i^{kji} - V_i^{kij}\right) dt &= 1dt \\ \lambda_i^{2,j}\left(V_j^{kij} - V_j^{kji}\right) dt &= 1dt \end{aligned}$$

where $\lambda_j^{2,i}$ is the hazard rate that player j will exit, when she is playing against player i and she would get the second prize, after player k has left. The left-hand side of the equation represents the benefits of staying, while the right-hand side represents the costs of staying. Because the values do not change over time, until a player exits, the hazard rate of exiting cannot change over time either. It is worth noticing that the strategy of player j is completely determined by the valuations of his opponent. In this case, due to asymmetry, the hazard rate of each player is not the same, but it can be easily computed: $\lambda_i^{2,j} = 1/(V_j^{kij} - V_j^{kji})$ and $\lambda_j^{2,i} = 1/(V_i^{kji} - V_i^{kij})$. In this case, we need to compute all six hazard rates for all the games with two players. This would be the case even without any restrictions on the parameters. The reason that we do not need to compute more than six is that there is no possibility that externalities will affect equilibrium behavior when there are only two players left. However, we need to compute all six. Unless in the asymmetric case without externalities, now when player i is facing player j the values for staying and for exiting of player j are different than when it is player k facing player j. Finally, there is some good news. Even in the most general case with any possible externalities, when we arrive to the final games with two players, all of the hazard functions are just the inverse of the differences in valuations, so they are all straightforward to compute.

It is also worth noticing that this equilibrium exhausts all the rents for all players. This means that the expected utility for player i of playing the two-player game against player j is just V_i^{kij}. That is, the utility player i would get if she just exits right away. Notice that now this value, is not independent of the identity of the opponent. The strategy and therefore the distribution of exit time is also not independent of hes opponent. Each player is imposing a negative externality on each other by staying in the game. The externalities are maximal in the sense that they fully dissipate all the rents for the other player. In the asymmetric case, with or without externalities, the larger the difference in valuations of my opponent, the lower is my exit rate, therefore the longer I stay and the larger is the externality I impose on my opponent. That is, in order to keep the value of playing the game constant, the larger is the differences in values of my opponent, the longer I need to stay to dissipate rents.

We can now focus on the game with three players. In this case the indifferent conditions for player i, j and k respectively are

$$\begin{array}{rcl} \lambda_j^3\left(W_i^{2,j}-V_i^{i\cdot\cdot}\right)dt+\lambda_k^3\left(W_i^{2,k}-V_i^{i\cdot\cdot}\right)dt & = & 1dt \\ \lambda_i^3\left(W_j^{2,i}-V_j^{j\cdot\cdot}\right)dt+\lambda_k^3\left(W_j^{2,k}-V_j^{j\cdot\cdot}\right)dt & = & 1dt \\ \lambda_i^3\left(W_k^{2,i}-V_k^{k\cdot\cdot}\right)dt+\lambda_j^3\left(W_k^{2,j}-V_k^{k\cdot\cdot}\right)dt & = & 1dt \end{array} \tag{24}$$

where $W_i^{2,j}$ is the continuation value for player i if player j exits. In the first equation, the first term in the left-hand side represents the hazard rate that player j will exit times the benefit to player i if that happens, the second term in the left-hand side represents the hazard rate that player k will exit times the benefit to player i if that happens and the right-hand side is the cost. The second and third equations are analogous. We are already using the assumption above to restrict the number of parameters. Without that assumption, the equilibrium would be more complicated to solve. The reason is that in the system 24 above we would have to write $W_i^{2,i}$ instead of $V_i^{i\cdot\cdot}$. That is, we would have to compute the continuation value for player i of receiving a prize in some time in the future, that would be determined by the strategies of other players. We would also have to think of whether the prize to player i is awarded at the time she exits or at the time the game ends. With the simplification we have to estimate one less parameter in a three-player game (although many more parameters in the games with more players), but conceptually is much easier to think about externalities in this setting.

Notice that now solving the game is not harder than in the asymmetric case. We have three equations with three unknowns but, unlike in the case with two players, there is more than one unknown in each equation. Moreover, due to asymmetry, in general the exit times will be different for each player. The system of equations now seems complicated. We need to solve for three variables $\lambda_i^3, \lambda_j^3, \lambda_k^3$ as a function of nine parameters, three in each equation. As we have seen before, the continuation game in the asymmetric case with externalities is not the same regardless of the identity of the player who exits. However, we can still compute the continuation value as the value of receiving the second prize, *i.e.*, $W_i^{2,j} = V_i^{jik}$. Therefore, the equilibrium can still be express as a function of the fundamentals only, but there are still six parameters to take into account. We can simplify the notation and write $\Delta_i^j \equiv V_i^{jik} - V_i^{i\cdot\cdot}$, that is, the continuation value of player i when player j exits.

$$
\begin{aligned}
\lambda_j^3 \Delta_i^j + \lambda_k^3 \Delta_i^k &= 1 \\
\lambda_i^3 \Delta_j^i + \lambda_k^3 \Delta_j^k &= 1 \\
\lambda_i^3 \Delta_k^i + \lambda_j^3 \Delta_k^j &= 1
\end{aligned}
\tag{25}
$$

We will not go into the details, but the solution to the system is

$$
\begin{aligned}
\lambda_i^3 &= \frac{\Delta_i^j \Delta_j^k - \Delta_j^k \Delta_k^j + \Delta_i^k \Delta_k^j}{\Delta_i^j \Delta_j^k \Delta_k^i + \Delta_i^k \Delta_k^j \Delta_j^i} \\
\lambda_j^3 &= \frac{\Delta_i^k \Delta_j^i - \Delta_k^i \Delta_j^i + \Delta_k^i}{\Delta_i^j \Delta_j^k \Delta_k^i + \Delta_k^j \Delta_i^k \Delta_j^i} \\
\lambda_k^3 &= \frac{\Delta_k^j \Delta_j^i - \Delta_i^j \Delta_j^i + \Delta_i^j \Delta_k^i}{\Delta_i^j \Delta_j^k \Delta_k^i + \Delta_k^j \Delta_i^k \Delta_j^i}
\end{aligned}
\tag{26}
$$

Notice that now the hazard rates are more complicated, and harder to solve. Moreover, it is now harder to provide an intuition for the formula. Similar to the asymmetric case without externalities, each hazard function is now a weighted average. Also, the denominator in each equation is the same.[15] The difference between the case with and without externalities is than in the case without externalities the continuation game has the same value regardless of the identity of the player who exits, *i.e.*, $\Delta_i^j = \Delta_i$. Therefore, when

[15]It can be shown that this equilibrium, where all players mix all the time, exist if and only if all the probabilities are positive. This happens whenever the difference in valuations for any given player is not very different from the differences in valuations of all the other players.

looking at the solution above, in the case without externalities we would have the same formula, but without superscripts in the Δ

$$\begin{array}{rcl} \lambda_i^3 & = & \frac{\Delta_i\Delta_j - \Delta_j\Delta_k + \Delta_i\Delta_k}{\Delta_i\Delta_j\Delta_k + \Delta_i\Delta_k\Delta_j} \\ \lambda_j^3 & = & \frac{\Delta_i\Delta_j - \Delta_k\Delta_j + \Delta_k}{\Delta_i\Delta_j\Delta_k + \Delta_i\Delta_k\Delta_j} \\ \lambda_k^3 & = & \frac{\Delta_k\Delta_j - \Delta_i\Delta_j + \Delta_i\Delta_k}{\Delta_i\Delta_j\Delta_k + \Delta_i\Delta_k\Delta_j} \end{array} \tag{27}$$

Now it is clearer to see the solution. Each term in the denominator is identical, and it is made of the three Δ for each of the three players. The terms in the numerator are all three possible combinations of pairs of two players, with the one with the negative sign corresponding to the player whose hazard rate it represents, *e.g.*, in the hazard rate for player i the only term with a negative sign is the one with the Δ for players j and k. Therefore, when we divide each term for its corresponding number, we get the expression in the system 22.

We have solved the system 24 above by solving for the values of λ as a function of Δ. This is what we need to do, from a theorist point of view, when we try to find the equilibrium strategies taken by each player, as a function of the parameters (values) that they observe. From the point of view of the econometrician, we still need to solve the system 24 above. However, from the econometrician point of view, we observe the hazard rate of exit (moreover, we observe the whole distribution of exit times), but we want to know the values of exiting for each player. Notice that the solution now is not exactly identified. We have six unknowns, but only three equations. Thus, this system is not identified in general, without further assumptions, that is, without specifying specific relations between the parameters. Since we have six parameters and three equations we need three more equations to exactly identify the parameter. The good news is that, as before, each Δ only appears in one equation of the system.

For example, one particular case is the case without externalities, as we have seen before. In that case the three extra equations are $\Delta_i^j = \Delta_i^k$, $\Delta_j^i = \Delta_j^k$ and $\Delta_k^i = \Delta_k^j$. What this implies is that we only have information on exit times, even if we have information on exit times for each player, we can estimate a model with asymmetries but no externalities. There are two ways that we can estimate externalities. The first one is using more data. If we have data on some of the V or some of the Δ we could solve the remaining. In this case with three players we would need data on at least three of the Δ in order to solve for the other three.

The second way is to use reduce the asymmetry and allow the asymmetric responses to be captured by the externalities. For example, we can assume that all players are identical and are affected by the other players in the same way, *i.e.*, $\Delta_i^j = \Delta^j$. This way the system above becomes

$$\begin{aligned}
\lambda_j^3\Delta^j + \lambda_k^3\Delta^k &= 1 \\
\lambda_i^3\Delta^i + \lambda_k^3\Delta^k &= 1 \\
\lambda_i^3\Delta^i + \lambda_j^3\Delta^j &= 1
\end{aligned} \tag{28}$$

due to the strong symmetry, each of the products in this system have to be equal to one half, i.e., $\lambda_i^3\Delta^i = \lambda_j^3\Delta^j = \lambda_k^3\Delta^k = 0.5$. In this case the parameters of interest are just half of the inverse of the hazard rate, *i.e.*, $\Delta^i = \frac{1}{2\lambda_i^3}$. This is in line with our interpretation above regarding externalities, Δ^i measures the externality that player i exerts on the other players when she exits.

C Identification and Externalities

In this section, we discuss in details the identification in symmetric and asymmetric games, as well as in games with externalities. We show that games without externalities are always identified, whereas games with externalities are only identified with restrictions in the parameter space.

C.1 Symmetric Game

For simplicity and easy of exposition we will first layout out how to estimate the symmetric game first. The results will also help us as a benchmark to compare with the results with an asymmetric and a game with externalities. Let consider a game with n identical players without externalities. In the first stage of the game, the players face the following trade-off

$$\sum_{j\neq i}\eta_j^n(t)dt\prod_{l\neq i,l\neq j}(1-p_l)\Delta^n(t) = \prod_{l\neq i}(1-p_l)dt$$

Therefore, the solution is given by

$$\sum_{j\neq i}\eta_j^n(t)\Delta^n(t) = 1$$

Since $\Delta^n(t) \equiv W^{n-1}(t) - V^n(t)$ is the same for all players, we have $\eta_j^n(t) = \eta^n(t)$. Moreover, the equilibrium here is determined by

$$\sum_{j\neq i}\eta_j^n(t)\Delta^n(t) = (n-1)\eta^n(t)\Delta^n(t) = 1$$

Thus,

$$\eta^n(t) = \tfrac{1}{(n-1)\Delta^n(t)} \tag{29}$$

$$\Delta^n(t) = \tfrac{1}{(n-1)\eta^n(t)} \tag{30}$$

Where equation 29 defines the equilibrium strategies as a function of the fundamentals of the game, and equation 30 defines the parameters to be estimated as a function of the observed behavior. Notice that although we could estimate $\Delta^n(t)$ for each game with n players, $\Delta^n(t)$ is not a function of the fundamentals of the game only, since it contains $W^{n-1}(t)$ which is the continuation value of a symmetric game with $n-1$ players. However, in a game with $n = 2$ players, since the game ends when one of the players exit, the value of $W^{n-1}(t)$ is just the value of getting the last "prize" that is the price the city pays to the last farmer. In other words, $W^1(t) = V^1(t)$. Therefore, in the last game we can recover $\Delta^2(t) \equiv V^1(t) - V^2(t)$, which is a function of the fundamentals of the game. Once we estimate this, we can compute the expected value of playing a game with two players, and put this value into the continuation value of a game with three players and so on. Notice that without information on the prices the city paid to the farmers, we cannot identify $V^1(t)$ and $V^2(t)$ independently. We could identify $V^3(t)$, $V^4(t)$ and so on, but not the first two because after the $(n-1)^{th}$ player exits, the n^{th} exits immediately after.

C.1.1 Example: Symmetric game with three players

In this example there are only three players and the game is fully characterized by the three parameters V^1, V^2, V^3 and $v(t)$. Moreover, because of symmetry, the games with two players are all identical. Thus, for each game, we will have two exit observations: one corresponding to a symmetric game with two players and another one corresponding to a symmetric game with three players. For this case, equation 30 corresponds to

$$\Delta^3 \equiv (V^2 - V^3) = \tfrac{1}{2\eta^3} \quad \text{and} \quad \Delta^2 \equiv (V^1 - V^2) = \tfrac{1}{\eta^2}$$

That is, in a symmetric game, players are identical and we only have one hazard rate of exit in each game. Thus, with two games, we can recover one parameter from each of the two games. If we have many games in the data, but only have information on exit times, we can recover $\Delta^3 \equiv V^2 - V^3$ and $\Delta^2 \equiv V^2 - V^3$, and the shape of $v(t)$ given parametric assumptions. Because the game is symmetric, there is no need to discuss other variables or additional data to estimate parameters.

C.2 Asymmetric Game

Although the symmetric game is easy to estimate and it only requires data on exit times, most settings would require the game to be asymmetric. We now consider a game with n asymmetric players without externalities. In the first stage of the game, the players face the following trade off

$$\sum_{j\neq i}\eta_j^n(t)dt\prod_{l\neq i,l\neq j}(1-p_l)\Delta_i^n(t)=\prod_{l\neq i}(1-p_l)dt$$

Therefore, the solution is given by

$$\sum_{j\neq i}\eta_j^n(t)\Delta_i^n(t)=1$$

Notice that now $\eta_j^n(t)$ is not the same across players. Moreover, with the assumption that $\Delta_i^n(t)=\Delta_i^n\cdot v(t)$, we can solve the system and get

$$\eta_j^n\left(t\right)=\frac{1}{v\left(t\right)}\left[-\frac{1}{\Delta_i^n}\left(\frac{n-2}{n-1}\right)+\frac{1}{n-1}\sum_{j\neq i}\frac{1}{\Delta_j^n}\right] \tag{31}$$

$$\Delta_i^n=\frac{1}{v\left(t\right)\left[\sum_{j\neq i}\eta_j^n(t)\right]} \tag{32}$$

Where equation 31 defines the equilibrium strategies as a function of the fundamentals of the game, and equation 32 defines the parameters to be estimated as a function of the observed behavior. Notice that the symmetric case is a particular case of the asymmetric case because when $\Delta_i^n=\Delta_j^n$ for all farmers, then equation 31 above simplifies to equation 29. Also, it is worth noticing that whereas the behavior of each player depends on the valuations of all players in a non-linear way, as shown in equation 31, to recover the valuation of a given player, we just need to add up the instantaneous probability of exiting of all the other players, as shown in equation 32. Moreover, because the shape of $\Delta_j^n(t)$ is the inverse of the shape of $\eta_j^n(t)$ we can define $\eta_j^n(t)\equiv\eta_j^n/v\left(t\right)$, and get

$$\Delta_i^n = \frac{1}{\sum_{j \neq i} \eta_j^n} \tag{33}$$

Although we could estimate $\Delta_i^n(t)$ for each farmer for each game with n players, $\Delta_i^n(t)$ is not a function of the fundamentals of the game only, since it contains $W_i^{n-1}(t)$ which is the continuation value of an asymmetric game with $n-1$ players. However, following the same argument as before, in a game without externalities we have that $W_i^1(t) = V_i^1(t)$. Therefore, we could identify all $V_i^n(t)$ except the first two because after the $(n-1)^{th}$ player exits, the n^{th} exits immediately after.

C.2.1 Example: Asymmetric game with three players

In this example there are only three asymmetric players, and the game is characterized by nine parameters: $V_i^3 < V_i^2 < V_i^1$, for $i = 1, 2, 3$. Where V_i^l is the value that player i gets from getting the l^{th} prize. Because of the asymmetry, there are three different games with two players. Equation 32 corresponds to three equations when there are three players

$$\Delta_1^3 \equiv (V_1^2 - V_1^3) = \tfrac{1}{\eta_2^3+\eta_3^3} \quad \text{and} \quad \Delta_2^3 = \tfrac{1}{\eta_1^3+\eta_3^3} \equiv (V_2^2 - V_2^3) \quad \text{and} \quad \Delta_3^3 = \tfrac{1}{\eta_2^3+\eta_1^3} \equiv (V_3^2 - V_3^3) \ ;$$

two equations when player one exits first

$$\Delta_2^{2,1} \equiv \left(V_2^1 - V_2^2\right) = \frac{1}{\eta_3^{2,1}} \text{ and } \Delta_3^{2,1} \equiv \left(V_3^1 - V_3^2\right) = \frac{1}{\eta_2^{2,1}};$$

two equations when player two exits first

$$\Delta_1^{2,2} \equiv \left(V_1^1 - V_1^2\right) = \frac{1}{\eta_3^{2,2}} \text{ and } \Delta_3^{2,2} \equiv \left(V_3^1 - V_3^2\right) = \frac{1}{\eta_1^{2,2}};$$

and two equations when player three exits first

$$\Delta_1^{2,3} \equiv \left(V_1^1 - V_1^2\right) = \frac{1}{\eta_2^{2,3}} \text{ and } \Delta_2^{2,3} \equiv \left(V_2^1 - V_2^2\right) = \frac{1}{\eta_1^{2,3}}.$$

Where η_i^3 is the hazard rate of player i when there are three players in the game and $\eta_i^{2,j}$ is the hazard rate of player i when there are two players in the game and player j has already exited. Two important things to remark here. First, looking at the equations for the game with three players, is clear that we cannot recover V_i^2 from V_i^3 for any of the players, without additional information. Thus we need a normalization, typically we normalize

$V_1^3 = V_2^3 = V_3^3 = 0$. Second, looking at the six equations for the three games with two players, we can see that there are only three independent equations. For example, we have that $\Delta_2^{2,1} = (V_2^1 - V_2^2) = \Delta_2^{2,3}$. This is because, in a game without externalities the differences in valuations between staying and exiting is independent of the identities of the players who already exited. In terms of identification, this implies that $\eta_3^{2,1} = \eta_1^{2,3}$. That is, the hazard rate of player 3, when playing against player 2 (because player 1 exited), is the same as the hazard rate of player 1, when playing against player 2 (because player 3 exited). This is important because it means that an asymmetric game without externalities is over-identified, but also that we can compute these hazard rates empirically and use them as a test for externalities.

In contrast to the symmetric game, when players are asymmetric, we can recover a hazard rate from each player in each game. That means we can recover three hazard rates from the game with three players and two hazard rates from each of the three asymmetric games. This is a total of nine hazard rates. However, as mentioned above, three of the hazard rates in the two-player games as redundant, so we end up with only six independent equations, and we need to normalize three parameters, one for each player. If we normalize $V_1^3 = V_2^3 = V_3^3 = 0$, then we can identify $V_i^2 < V_i^1$, for $i = 1, 2, 3$.

C.3 Game with externalities

We now consider a game with n players with externalities. In a given stage of the game, when there are n players in the game, the players face the following trade-off

$$\sum_{j\neq i} \eta_j^n(t)dt \prod_{l\neq i, l\neq j}(1-p_l)\Delta_i^{n,j}(t) = \prod_{l\neq i}(1-p_l)dt$$

Therefore, the solution is given by

$$\sum_{j\neq i} \eta_j^n(t)\Delta_i^{n,j}(t) = 1$$

Notice that now $\eta_j^n(t)$ is not the same across players. Moreover, because of the externalities, the continuation value for player i after player j exits, depends on the identity of player j. Notice that $\Delta_i^{n,j}(t)$ is the difference between staying and exiting for player i, when staying means that player j would quit, in a game with n players at time t. With the assumption that $\Delta_i^{n,j}(t) = \Delta_i^{n,j} \cdot v(t)$, we can see that, as in the asymmetric case, the shape of $\eta_i^n(t)$ would be equal to the inverse of $v\,(t)$. However, in general we might not have a closed-form solution for η_j^n, but we know it is the solution to a linear system of equations. We can get

$$\sum_{j\neq i}\eta_j^n(t)\Delta_i^{n,j}(t) = 1 \tag{34}$$

$$\Delta_i^n = \frac{1}{v\left(t\right)\left[\sum\limits_{j\neq i}\eta_j^n(t)\right]} \tag{35}$$

Where equation 34 defines the equilibrium strategies as a function of the fundamentals of the game (implicitly), and equation 35 defines the parameters to be estimated as a function of the observed behavior. It is worth noticing that whereas the behavior of each player depends on the valuations of all players in a non-linear way, as shown in equation 34, to recover the valuation of a given player, we just need to add up the instantaneous probability of exiting of all the other players, as shown in equation 35. Moreover, because the shape of $\Delta_i^{n,j}(t)$ is the inverse of the shape of $\eta_i^n(t)$ we can define $\eta_i^n(t) \equiv \eta_i^n/v\left(t\right)$, and get

$$\Delta_i^n \equiv \sum_j \Delta_i^{n,j} = \frac{1}{\sum\limits_{j\neq i}\eta_j^n} \tag{36}$$

In general, as we illustrate with the example below we cannot estimate $\Delta_i^{n,j}(t)$ for each farmer for each game with n players, because there are more elements $\Delta_i^{n,j}(t)$ than observations in the data. $\Delta_i^{n,j}(t)$ is not a function of the fundamentals of the game only, since it contains $W_i^{n-1,j}(t)$ which is the continuation value for player i of a game with externalities with $n-1$ players, when player j exited first. In this case, in a game without externalities, we cannot identify all the parameters. Nonetheless, in practice, externalities will have a particular form, which means that there are restrictions on the values of $\Delta_i^{n,j}$, with enough restrictions, we could identify all the parameters in the game.

C.3.1 Example: Game with externalities with three players

In this example there are only three players, and the game is characterized by eighteen parameters, six for each player: $V_i^{123}, V_i^{132}, V_i^{213}, V_i^{231}, V_i^{312}, V_i^{321}$, for $i = 1, 2, 3$. Where V_i^{xyz} is the value that player i gets when the allocation of prizes is such that player x gets the first prize, player y gets the second prize and player z gets the third prize. In the case we are considering here. However, we can restrict the number of parameters by having some restrictions on the vector of preferences for each player. Because of the asymmetry, there are three different games with two players.

Before we show the equations, notice that in the first game, with three players, no player has exited yet. However, in the second round, one player would have already exited and, because of the externalities, the continuation value of player i of staying would depend on the identity of the player that exited. We can define V_i^2 as the expected value for player i of playing a game with two players. Taking as given the strategies for the players, we can compute this "average" value. Conditional on player i not exiting in the first round, she will face player j with probability $\frac{\eta_k^3}{\eta_j^3+\eta_k^3}$ and face player k with probability $\frac{\eta_j^3}{\eta_j^3+\eta_k^3}$. If player k exits, then player i faces player j then her valuation would be V_i^{kij}, and if player j exits, then player i faces player k and her valuation would be V_i^{jik}.[16] Then, we can define

$$V_i^2 \equiv \frac{\eta_k^3}{\eta_j^3+\eta_k^3} V_i^{kij} + \frac{\eta_j^3}{\eta_j^3+\eta_k^3} V_i^{jik}. \tag{37}$$

Using a similar argument, we can define V_i^3 as the expected value for player i of exiting first in a game with three players as

$$V_i^3 \equiv \left(\frac{\eta_j^{2,i}}{\eta_j^{2,i}+\eta_k^{2,i}} V_i^{ijk} + \frac{\eta_k^{2,i}}{\eta_j^{2,i}+\eta_k^{2,i}} V_i^{ikj} \right). \tag{38}$$

We have expressed the expected values as functions of the valuations and the hazard rates, but as we show below, all the hazard rates can be written as a function of the valuations, so the expected valuations can be written as a function of the valuations only. We can now write the corresponding equations: three equations when there are three players

$$\Delta_1^3 \equiv (V_1^2 - V_1^3) = \tfrac{1}{\eta_2^3+\eta_3^3} \quad \text{and} \quad \Delta_2^3 = \tfrac{1}{\eta_1^3+\eta_3^3} \equiv (V_2^2 - V_2^3) \quad \text{and} \quad \Delta_3^3 = \tfrac{1}{\eta_2^3+\eta_1^3} \equiv (V_3^2 - V_3^3) \ ;$$

two equations when player one exits first

$$\Delta_2^{2,1} \equiv \left(V_2^{132} - V_2^{123}\right) = \frac{1}{\eta_3^{2,1}} \text{ and } \Delta_3^{2,1} \equiv \left(V_3^{123} - V_3^{132}\right) = \frac{1}{\eta_2^{2,1}};$$

two equations when player two exits first

[16] Even in a game with externalities, when there are only two players left, there are only two possible valuations for each player. In other words, conditional on a particular exit history, externalities play no role when there are only two players left. In a game with externalities when player k has exited, player i would get a value of V_i^{kij} if she exits, or V_i^{kji}if player j exits, with $V_i^{kji} > V_i^{kij}$. Playing this 2-player game has a expected value of V_i^{kij} for player i.

$$\Delta_1^{2,2} \equiv \left(V_1^{231} - V_1^{213}\right) = \frac{1}{\eta_3^{2,2}} \text{ and } \Delta_3^{2,2} \equiv \left(V_3^{213} - V_3^{231}\right) = \frac{1}{\eta_1^{2,2}};$$

and two equations when player three exits first

$$\Delta_1^{2,3} \equiv \left(V_1^{321} - V_1^{312}\right) = \frac{1}{\eta_2^{2,3}} \text{ and } \Delta_2^{2,3} \equiv \left(V_2^{312} - V_2^{321}\right) = \frac{1}{\eta_1^{2,3}}.$$

Where η_i^3 is the hazard rate of player i when there are three players in the game and $\eta_i^{2,j}$ is the hazard rate of player i when there are two players in the game and player j has already exited. Two important things to remark here. First, looking at the equations for the game with three players, is clear that we cannot recover V_i^2 from V_i^3 for any of the players, without additional information. Thus we need a normalization. In the case without externalities, typically we normalize $V_1^3 = V_2^3 = V_3^3 = 0$. Notice that, because of the externalities, V_i^2 and V_i^3 are not fundamentals, but functions of the fundamentals, so one normalization for each player, would just reduce the parameters to be estimated by three from eighteen to fifteen. In many cases, however, we can assume that after one player exits, his valuation is not affected by the subsequent exit order. This assumption implies that $V_i^{ijk} = V_i^{ikj}$ for each player, which reduces the number of parameters by three. The two normalizations mentioned above means that $V_1^{123} = V_1^{132} = V_2^{231} = V_2^{213} = V_3^{312} = V_3^{321} = 0$, that is, whenever any player exit first, her value is zero, regardless of what the other players do. Second, looking at the six equations for the three games with two players, we can see that now all six equations are independent, unlike in the case without externalities. For example, we have that $\Delta_2^{2,1} = (V_2^{132} - V_2^{123}) \neq (V_2^{312} - V_2^{321}) = \Delta_2^{2,3}$. In other words, the value of playing a game with two players for player 2 is different depending on the identity of the other player.

Without any normalization we have nine independent equations and eighteen parameters, so we cannot identify all the parameters. With the two normalizations mentioned above, we can identify the three expected values V_1^2, V_2^2 and V_3^2. In fact, we can identify all three just with the first three equations for the game with three players. However, each expected values is a function of two of the eighteen valuations, which are not independently identified, without further restrictions. For example, with the normalizations we know that $V_1^2 = \frac{1}{\eta_2^3+\eta_3^3}$. However, we know that $V_1^2 \equiv \frac{\eta_3^3}{\eta_2^3+\eta_3^3} V_1^{312} + \frac{\eta_2^3}{\eta_2^3+\eta_3^3} V_1^{213}$, and V_1^{312} and V_1^{213} are not identified in general, because we have nine equations and twelve remaining parameters to be identified.